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ABSTRACT

One of the strengths of the Pennsylvania Council of Teachers of Mathematics (PCTM) is that it gives mathematicians and mathematics educators the opportunity to exchange and contribute to each other's professional growth. The topic for each yearbook is chosen to coincide with the annual PCTM meeting. This 1986 yearbook contains 17 articles related to ideas, suggestions, practices, and programs that have the potential to improve the teaching and learning of mathematics. Topics include: improving academic achievement; the use of concrete examples; Euclidean concepts; the use of daily sequences; how to improve mathematics skills; using mathematics education research to improve instruction; activities involving the Statue of Liberty; whole brain thinking; using models; using computers to teach mathematics; an example of an innovative high school curriculum; Monte Carlo methods; graphing with a microcomputer; process learning in mathematics; developing an achievement test; computer resource centers; and evaluating computer software. (CW)

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MATHEMATICS TEACHING AND LEARNING

1986 Yearbook

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MATHEMATICS TEACHING AND LEARNING

1986 Yearbook

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Co-Editors

PENNSYLVANIA COUNCIL OF TEACHERS OF MATHEMATICS

PREFACE

One of the strengths of the Pennsylvania Council of Teachers of Mathematics is that it gives mathematicians and mathematics educators across Pennsylvania opportunities to exchange ideas and contribute to each other's continuing professional growth. The organization attempts to facilitate such exchanges through an active program of publications and conferences.

The 1986 Yearbook — MATHEMATICS: TEACHING AND LEARNING — is the second PCTM yearbook to be developed and distributed to the membership. The theme was chosen to be congruent with that chosen for the 35th Annual Meeting of the organization. In fact, a number of the authors of articles in this yearbook are on the Conference program.

The articles in MATHEMATICS: TEACHING AND LEARNING focus on ideas, suggestions, practices and programs that have the potential to improve the teaching and learning of mathematics in Pennsylvania's schools. Articles deal with topics of interest to elementary teachers, secondary mathematics teachers, college mathematics teachers, teacher educators, mathematics supervisors, and curriculum coordinators. They were written by practitioners and researchers from both basic and higher education who responded to a call for manuscripts which was sent to PCTM members in Spring, 1985.

Considerable thanks go to a number of people for their important contributions to the 1986 Yearbook. Suzanne Harpster, Denise Brooks, Janet Bobango and Helen Fiber, all at The Pennsylvania State University, provided valuable editorial assistance. They helped us to ferret out the aberrations that invariably appear in the publishing process. The authors of the manuscripts deserve considerable credit for taking the initiative and the time to place their ideas in front of their peers. The commercial and institutional advertisers also deserve our thanks for their willingness to invest their money by buying space in the yearbook. Last but not least, the PCTM Executive Board deserves credit for its continuing support of the efforts of the Publications Committee.

We are glad to have had the opportunity to serve as editors for the 1986 Yearbook. We hope that the readers will carefully read the articles in MATHEMATICS: TEACHING AND LEARNING and implement the ideas that are relevant for them. We invite response from the readers, authors and advertisers.

Robert F. Nicely, Jr.
The Pennsylvania State University

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University of Pittsburgh at Johnstown
1986 Yearbook Editors

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IMPROVE ACHIEVEMENT BY INCREASING INTENSITY AND DECREASING DOWN TIME

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Many attempts to improve achievement in the schools end in frustration. Not frustration caused by student apathy or teacher ineptness but rather frustration from the overwhelming number of available research studies and innovative programs from which to select. This is particularly true in the area of mathematics.

There are curricular and instructional resources such as mainframes, micros, turtles, computer assisted instruction, diagnostic/prescriptive mathematics, games, puzzles, calculators, memory techniques and problem-solving programs. There are learning styles to be attended to. Hemispheric lateralization (left/right brain), Jung's Theory of Psychological Types (measured by the Myers-Briggs Type Indicator) and Dunn's learning styles (which include reclining, eating and listening to music while studying) are just a few. A third factor is the prominent role played by state, national and international testing programs, some of which are in the realm of competency tests where student remediation is mandated. The number of combinations of programs, learning styles and tests is astronomical. It is little wonder that many school improvement attempts end in frustration.

There are, however, several common threads which wind through a number of the research studies and data from state and international testing programs. These relate to "intensity of instruction" and reduction of "down time."

Down Time

The Second International Mathematics Study (SIMS), completed in 1982, not only measured the attained curriculum (mathematics achievement) in grades 8 and 12 but also collected information on the intended curriculum (scope and sequence) and the implemented curriculum (to what extent and using what methods were the various concepts taught). While data from the implemented curriculum may eventually prove to be more valuable than the achievement scores, it is, however, the achievement information or the "horse race" aspect of SIMS that gains the most attention.

In reference to achievement, little has changed since the First International Mathematics Study (FIMS) in 1964. Japan wins by a mile while the U.S. ranges from the international median to the bottom quartile of the twenty-two nation study. The United States may never equal Japan in this type of study, for a variety of reasons, most of which do not relate to

daily classroom instruction. Still there is much to be learned from comparing the implemented curriculum in Japan with the U.S., particularly in the area of intensity of instruction and the reduction of down time.

Table 1 shows the percent of time during a typical calculus or pre-calculus class allocated to each of four activities. Japanese teachers spend nearly half (48%) of the period introducing new content while their U.S. counterparts use 40% of the available time for this purpose. Reviewing previous work consumes one-third (32%) of the Japanese classrooms compared to one-fifth (20%) for the U.S. The remaining two categories are "down time" and it is here that differences are startling. The U.S. calculus/pre-calculus teachers have, on the average, forty percent "down

TABLE 1
PERCENT OF CLASS TIME SPENT ON VARIOUS ACTIVITIES

POPULATION B
(GRADE 12 — Calculus/Pre-Calculus)

	Japan	*USA*
New Content	48%	40%
Review	32%	20%
Supervised Study	14%	30%
Management	6%	10%
Students Responding in Each Class	67%	14%

time" in every class, twice as much as in Japan. If supervised study and management translates into little or no instruction, then the equivalent of two days out of every five are non-teaching days. In addition, U.S. teachers spend an average of fifty minutes a week testing. With one day spent testing, and forty percent of the remaining time "down time," then only a little over the equivalent of two class periods per week are spent on instruction. In fact, the data show about one hundred minutes per week are used for actual instruction. If the supervised study category actually consisted of some type of peer tutoring or team learning, then it should not be labeled "down time," but this is not the case.

Two of the classic studies on "time on task" were conducted by Jane Stallings in reading and Carolyn Evertson in mathematics. The Stallings study indicated that in classrooms having high gains in reading achievement, teachers spent close to ninety percent of their time teaching. There was very little "down time." In contrast, the low/no gain classrooms received instruction for only about thirty-five percent of the class period, leaving sixty-five percent allocated to "down time." Here "down time" includes classroom management, reading silently, written assignments and social interaction.

The Evertson et al. study showed a similar pattern in mathematics classrooms. The more effective teachers used about half the period for

lecture, demonstration and discussion while the less effective teachers used only about eleven minutes of a fifty minute period for that purpose. The "down time" for the latter groups was nearly eighty percent of the class time! This may be a little misleading as an average of five minutes was used for small group instruction (2%), students working at the board (3%), self-paced instruction(5%) and testing (5%). However, this still leaves sixty-five percent of class time as "down time."

One of the significant outcomes of increasing actual instructional time is that more students have the opportunity to actively participate. Table 1 shows that sixty-seven percent of the students in Japan respond in each class while only fourteen percent respond in U.S. classes — and the class size in Japan is larger than in the U.S.

Intensity of Instruction

Table 2 shows SIMS pre-test scores, post-test scores, and the percent of classes covering each of two skills in the intervening time. In reference to the skill "addition of fractions with unlike denominators," in the typical U.S. eighth grade class, the pre-test score was forty-one percent correct. Ninety percent of the teachers subsequently taught it anyway. This

TABLE 2
SIMS PRE AND POST-TEST DATA FOR TWO ITEMS AND THE PERCENT OF CLASSES COVERING THE CONCEPT BETWEEN THE ADMINISTRATION OF THE PRE-TEST AND THE POST-TEST

POPULATION A (GRADE 8)			
Test Item: 1/3 + 2/5 =			
	Pre	*Post*	*% Covering in Class*
US	41	57	90
US (Algebra)	77	83	46
Japan	84	89	3
Test Item: (−2) − (+5) =			
	Pre	*Post*	*% Covering in Class*
US	21	40	93
US (Algebra)	45	71	92
Japan	18	72	98

instruction resulted in only a sixteen percent improvement in the achievement of the item — from forty-one percent correct to fifty-seven percent correct. The U.S. eighth grade algebra classes scored seventy-seven percent correct on the concept yet nearly half (46%) of the teachers taught it again, resulting in an improvement of six percent — from

seventy-seven percent correct to eighty-three percent correct. Pre-test scores in Japan indicated that eighty-four percent of the students had achieved the skill, so ninety-seven percent of the teachers did not teach it again (3% did) and still there was a five percent gain from pre-test to post-test.

For the skill related to fundamental operations with integers, the pattern was similar for the U.S. However, in Japan, since only eighteen percent of the students had mastered the concept on the pre-test, ninety-eight percent of the teachers taught the concept in the interim. The effects are evident from the resulting post-test scores of seventy-two percent correct. Their philosophy seems to be "if they know it, don't teach it again; if they don't know it, teach it with intensity." Interestingly, the U.S. outscored Japan on only two concepts — square root and division of fractions — both of which the Japanese considered of little importance.

Pennsylvania's state assessment program — Educational Quality Assessment — data support the intensity notion. The skill — addition with common fraction notation, unequal denominators — is mastered by about fifty-five percent of the eighth grade students and in grade eleven there is *no* significant improvement in this skill. There are other basic skills that show a similar pattern from eighth grade to eleventh grade, yet, surely many of these skills are included in ninth, tenth and eleventh grade mathematics programs.

Why does this happen? It is possible that the teacher's attitude might be, "I'd better spend a day or so on this skill since many of the students have not mastered it — but it really may be beyond their ability," while the student's attitude is, "I have had this every year since sixth grade but no one really expected me to be able to do it, so, I won't worry about learning it now." If so, intensity is lacking. There is no commitment. Minimum competency tests such as TELLS (Testing for Educational Learning and Literacy Skills — Pennsylvania's competency test), will have some effect on this scenario.

Implications for Instruction

What can be done, realistically, to improve achievement at a reasonable cost and with minimum in-service training? Increase the intensity of instruction and reduce "down time!" The following eight items provide a core of essential tasks for accomplishing this goal:

1. Teachers and instructional supervisors need to determine the specific core of concepts that are important and that students can master.
2. Teachers and students should be jointly responsible for student mastery of the core of skills and concepts.
3. Teachers and instructional supervisors need to identify and obtain materials that can be used to enhance student mastery of these skills.
4. Teachers and instructional supervisors need to develop a pool of test

items to measure mastery of the core concepts and skills and include a subset of these items on mid-term and final exams.

5. Teachers need to read selected journal articles related to "time on task" or reduction of "down time."
6. Instructional supervisors need to provide teacher in-service related to "time on task" or reduction of "down time."
7. Instructional supervisors need to use a supervisory model that will focus on "teaching time" vs. "down time," along with the number of students responding during each class.
8. Supervisors need to obtain and distribute materials and provide teacher in-service related to small group instruction, team learning and peer tutoring.

Other research studies and programs, particularly those related to learning styles, computer assisted instruction, memory techniques, and supervisory models have great potential, but for ease of implementation and cost effectiveness, strategies to address intensity of instruction and reduction of down time may provide the most efficient beginning.

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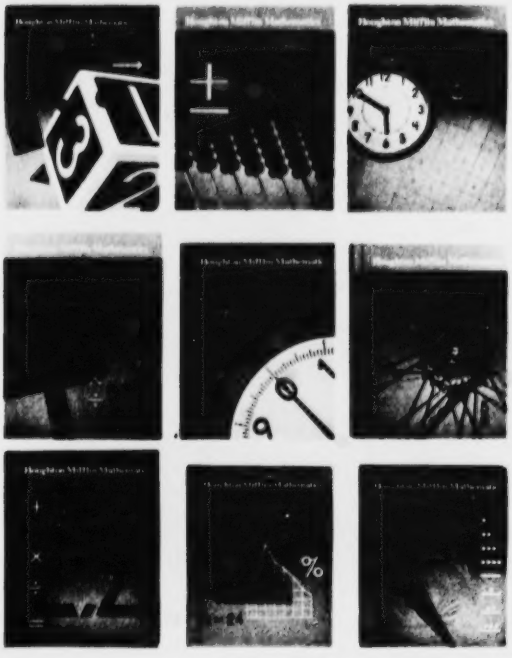

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CONCRETE CAN'T BE BEAT

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Johnstown, Pa. 15904

"Open your math book to page 42. Point to the first box..." If this sounds familiar to you — STOP — and consider the consequences. You may be creating an atmosphere in which math remediation will thrive. Many teachers have been introducing math concepts to primary school children in this abstract manner, when for literally decades experts have been suggesting that this is an ineffective method of approaching math — i.e., introducing or developing concepts without concrete or manipulative materials.

One of the most influential theoretical frameworks indicating the need to begin learning at a concrete level with young children is that of Jean Piaget. Piaget's theory of intellectual development suggests that there are fundamental differences between the mental abilities of children and adults. Children of ages approximately 6½ to 11 are functioning at what he calls the "concrete operational stage." (Piaget, 1953) During this stage children, although beginning to think logically, need concrete objects on which to reason. Therefore, the thinking of children is still limited in a sense because it is "concrete." With the help of concrete objects during this concrete operational period, children begin to depend less and less on manipulative objects as a way of learning. (Piaget and Inhelder, 1969)

Thus, the most important implication of Piaget's research for teaching mathematics at the primary level (K-3) is that children, who are for the most part in the preoperational and concrete operational stages, should be given ample opportunity to learn mathematics by doing — that is, by manipulating real objects. These hands-on experiences help a child acquire mental pictures needed to develop mathematical concepts. (Fuys and Tischler, 1979). In fact, complete mathematics curriculum guides have been developed for teachers on the basis of Piaget's stages of intellectual development. Other educators, such as Dienes and Bruner, stress the importance of engaging children in mathematics *actively*, rather than just watching and listening. (Dienes, 1960; Bruner, Goodnow & Austin, 1956).

The verdict is in — CONCRETE CAN'T BE BEAT! Therefore, this article is decreeing that from this day on all primary mathematics teachers *actively* involve their children with manipulative materials when introducing new concepts. The following ideas represent simple and inexpensive samples of concrete materials to introduce the concepts of place value and regrouping in addition and subtraction.

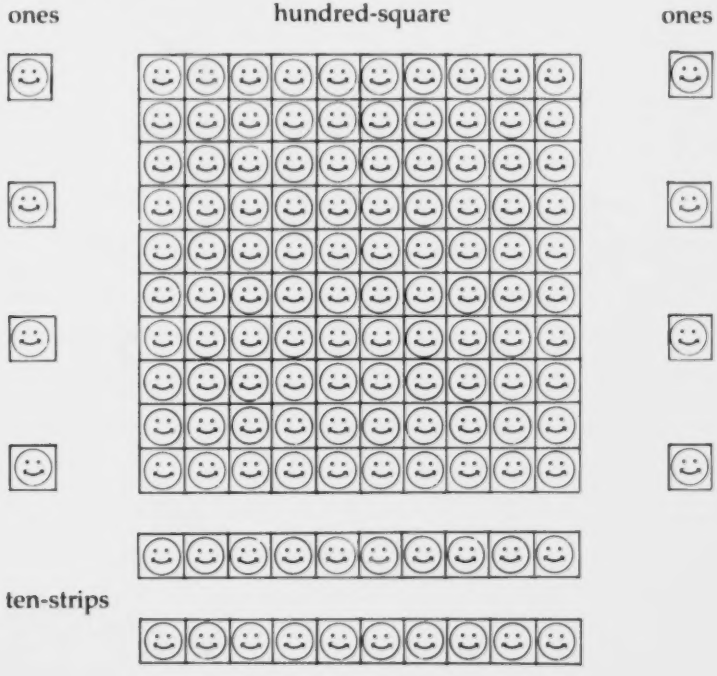
PLACE VALUE

Many teachers have been using Happy Faces to reward their students for a job well done. Those same Happy Faces can be used to demonstrate some of the concepts related to place value. A 10 x 10 array of Happy Faces should be duplicated so that each child in your class has at least 10 hundred squares, 20 ten-strips, and 20 ones.

The smallest pieces, the single Happy Faces, represent ones. They can be used as counters to develop rational counting, as well as in the early development of whole number operations.

The vertical or horizontal ten-strip contains 10 Happy Faces. This is easily verified by counting or by matching one-to-one. After being convinced that 1 ten equals 10 ones, children are willing to exchange 1 ten-strip for 10 ones or 10 ones for 1 ten-strip.

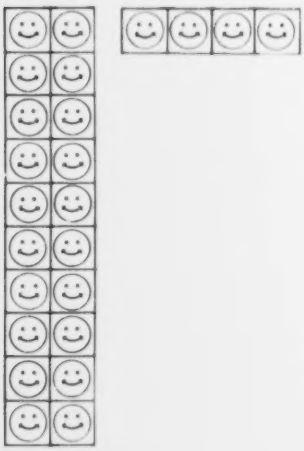
The 10 x 10 square contains 100 Happy Faces, which can be verified by the children. It also contains 10 ten-strips, and this can be checked by the children. After this verification, children are much more willing to accept an exchange of 1 hundred for 10 tens, or 10 tens for 1 hundred. The large hundred square, being a single sheet of paper, is more clearly identifiable as 1 hundred than are other place-value models. A bundle of 100 sticks is clumsy even when the sticks are grouped in 10 bunches of 10. (Troutman & Lichtenberg, 1982)



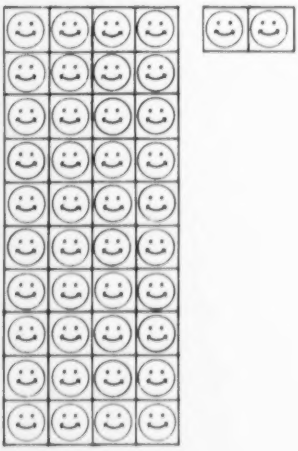
ACTIVITY 1 — Comparing 2 Two-digit Numerals Formed by the Reversal of Digits.

Using the Happy Face ten-strips and singles, children are able to distinguish between larger and smaller numerals by representing the number each represents.

Teacher: Show me 24.



Now show me 42.

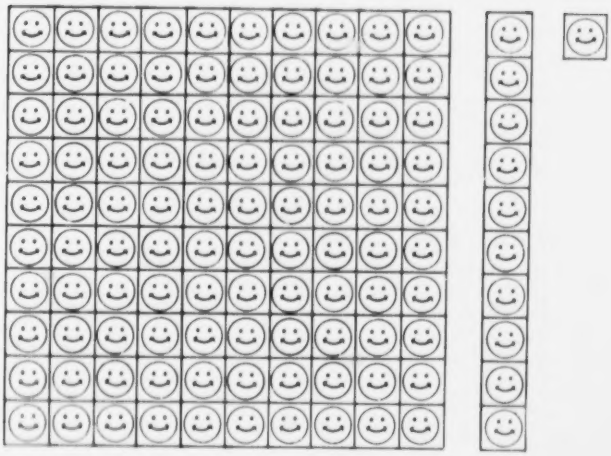


Teacher: Are 24 and 42 the same? Which is more? How can you tell which is more? Can you tell which number is greater without using your Happy Faces?

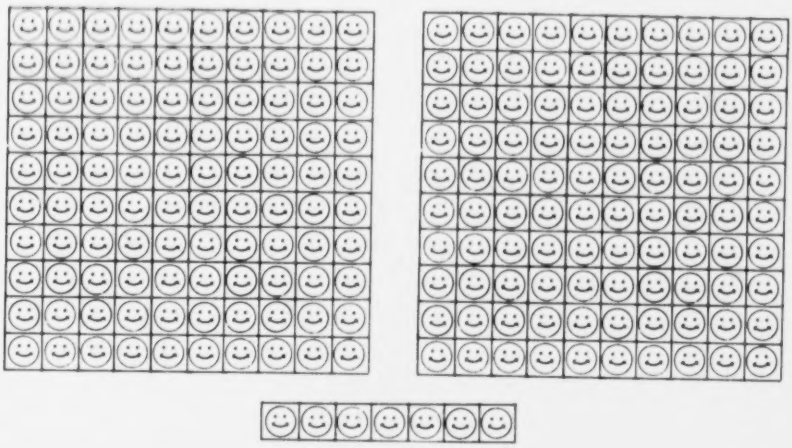
ACTIVITY 2 — Physically Representing Numbers Greater Than 100.

Children need the physical representation of numbers as a prerequisite to symbolic representation. Happy Faces are still appropriate and easy to manipulate.

Teacher: Show me 111.
How many hundreds?
How many tens?
How many ones?



Show me 207



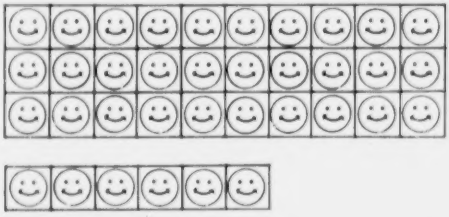
How many hundreds?
How many tens?
How many ones?
Which of the two numbers (111, 207) is larger?
How can you tell?

REGROUPING IN ADDITION AND SUBTRACTION

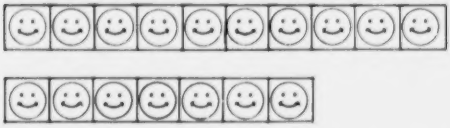
Don't throw away those Happy Faces. Once children have become familiar with their use in place value concepts, they can easily make the transition to regrouping in addition and subtraction.

ACTIVITY 1 — Addition with regrouping (36 + 17 =)

1. Show 36.



2. Show 17.

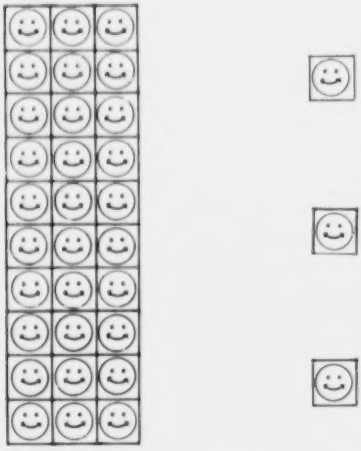


3. How many ones? 13 Can you trade 10 ones for a ten-strip? Yes. Now how many ones? 3 ones.
4. With the ten-strip you got from the 10 ones and the ten-strips you already had — now how many tens do you have? 5 tens.

5 tens + 3 ones = 53

ACTIVITY 2 — Subtraction with regrouping (33 − 25 =)

1. Show 33.



2. We want to subtract 25. Can we do this without a trade-in? No.

3. Exchange 1 ten-strip for 10 ones.
4. Now how many ones do you have? 13 ones.
5. How many tens do you have? 2 tens.
6. Can you subtract 5 ones? Yes.
   How many ones are left? 8 ones.
7. Can you subtract 2 tens? Yes.
   How many tens are left? 0 tens.
8. So 0 tens and 8 ones is _____ .

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PRECURSORS TO EUCLIDEAN CONCEPTS

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Geometric concepts and terms are an integral part of mathematical education. Teachers need to know about the development of spatial concepts and children's labeling of geometric forms so that they can provide appropriate learning experiences, This article summarizes the sequences in which children acquire the spatial concepts (topological → Euclidean & projective) and labels for geometric forms (metaphoric → proto-Euclidean → technical) and presents systems for categorizing triangles and quadrilaterals that can serve as advance organizers for lessons using materials and activities directed towards learning Euclidean geometry.

Space: Topological → Euclidean & Projective

When introducing basic concepts of informal geometry to children, it is important to approach the subject from within a topological framework (Copeland, 1984; Heimer & Trueblood, 1978). This belief derives from the theories of Piaget which maintain that a child's first concepts of space are topological, not Euclidean or projective (Piaget & Inhelder, 1956; Piaget, Inhelder, & Szeminska, 1960). Activities for children based on this approach intend to enhance their geometric thinking and the eventual construction of projective and Euclidean concepts of space (e.g., D'Augustine, 1966; Heimer & Trueblood, 1978; Saunders & Bingham-Newman, 1984).

Teachers can observe and foster children's interest and development in understanding topological space in five ways:

(1) *Enclosure* — the ability to differentiate closed from partly closed spaces (e.g., "open," "closed," "in," "out," "inside," "outside," "edge," "border");

(2) *Separation* — the ability to deal with part-whole relations: (a) dividing and reconstructing wholes (e.g., doing puzzles); (b) using comparable parts to make different wholes (e.g., using same set of blocks to make different products); (c) using different parts to make comparable wholes (e.g., using five large blocks to make same sized wall as one made with 20 small blocks); (d) realizing that what is whole and what is part is relative (e.g., a doll house embedded within a child's bedroom is a part of the bedroom, but in relation to the toys inside, it is a whole);

(3) *Proximity* — the ability to make judgments of distance (e.g., "near to," "far," "next to," "between," "on"): (a) skill in moving through space; (b) skill in moving objects in relation to one another (e.g., making a bridge or ramp with blocks);

(4) *Order* — the ability to maintain direction and a consistent sequence (e.g., "next to," "between");

(5) *Continuity* — the ability to view space as continuous so that several routes lead to the same point: (a) recognition that a circuitous route may lead to the same end point as a straight line; (b) development and use of alternative paths for objects and self movements.

Topological geometry is referred to as "rubber sheet" or "balloon" geometry. Relations and properties of points, lines, and regions that not altered by expansion, tilting, or compression are topological. Consider a group of drawings on a balloon. When the balloon is inflated, squares may begin to look like circles and lines may move further apart. Still lines that previously did not intersect remain separated; continuous lines remain continuous. Properties of the space that do not change when the balloon is twisted, stretched or compressed are called topological properties. Concepts of Euclidean space, on the other hand, include degree of angularity, absolute length, direction of curvature, and directionality within a coordinate system. These Euclidean properties are affected in the "rubber sheet" example above.

Children begin to understand properties of projective space (properties that are the same for an object and for the shadow it casts) when they begin to notice differences in viewpoints from different positions in space and begin quantifying direction and distance. Piaget, Inhelder, and Szeminska (1960) state that, "the elaboration of reference systems, the conservation of distance, and the grouping of changes of position all occur at the same level of development, and together they bring about the transition from topological to Euclidean notions of space (p. 391.)."

Labeling: Metaphoric → Proto-euclidean → Technical.

It is equally important for teachers to understand the development of children's language for geometric shapes.

When first attempting to name shapes, children employ metaphoric modes for conceptualizing and verbalizing geometric shapes. Geometric figures are described in terms of their resemblance to familiar concrete objects. For instance, children identify a rectangle as a "building," a trapezoid as a "handkerchief with one corner cut off," or a triangle as a "sailboat." Children respond concretely to the stimulus and use a familiar word which seems to apply. Children do not analyze or respond to distinctive features of the shape, but only to the shape as a whole. Combining shapes creating new patterns are likewise responded to as a whole, while rotating shapes in space causes children to see a different form and respond accordingly. For example, children who say "box" to a square say "diamond" to a square which is rotated.

Proto-euclidean modes of labeling geometric forms are similar to the metaphoric in being "polarized." A "polarized" geometry is one in which the properties of the figures vary as a function of their orientation in the

spatial field. Proto-euclidean labeling, however, is different from metaphoric labeling in that shapes are described primarily in terms of their "sides" and "angles" rather than in terms in their resemblance to familiar concrete objects. Children first learn the names of squares and triangles. If the sides and angles of the figures are the same, the square is a "perfect square" and the equilateral triangle is a "perfect triangle." The generic square, however, is defined by *either* four equal sides *or* "some" right angles. Children generate different shapes from squares and triangles and give them different names by performing operations on the generic concept:

(1) *Long/Tall* — sides not all equal (e.g., rectangle = "long" square);

(2) *Slanty* — not all sides meet at right angles (e.g., rhombus = "slanty" square);

(3) *Slanted* — axes-of-symmetry of the figure-as-a-whole are not parallel to the axes of vision (e.g., ◇ = slanted, long square); and

(4) *Half-a* — some portion of a square is transsected and removed (e.g., right angle = "half-a-square").

At this level some transformations are polarization-specific or rotation-sensitive: long/tall and slanted. Proto-euclidean labeling is a synthesis of and a transition between the metaphoric and Euclidean mode of thinking and verbalizing about geometric forms.

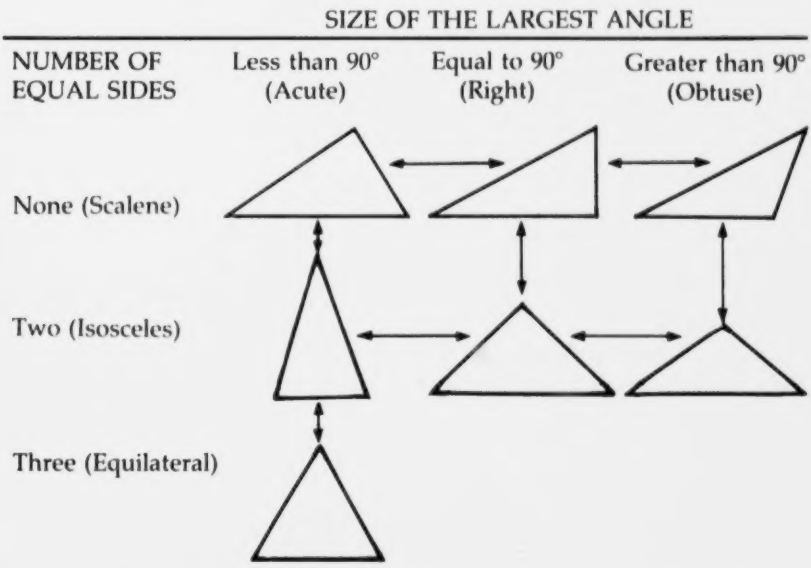

FIGURE 1. Geometric Technical Taxonomy: 3-Sided Figures

Categorizing of Triangles and Quadrilaterals.

A geometry is a formal set of transformational rules for generating any given figure (i.e., any array of connected points in a spatial system) from any other figure in the same spatial system. It is important for children to experience change within objects as well as to see differences between objects to prepare for learning the relationships between and within the categories of triangles and the quadrilaterals. Teachers who are aware of this are in a better position to plan and implement lessons and activities leading to children's learning and remembering the names of various triangles and rectangles.

It is possible to name seven different types of triangles. The system for categorizing 3-sided figures is based upon two separate and co-ordinate factors: (1) size of largest angle and (2) number of equal sides (See Fig. 1). With the exception of the equilateral triangle which by definition must be acute, each type of triangle is represented by the intersection of these two dimensions. All instances of three-sided figures in this system are equally specific.

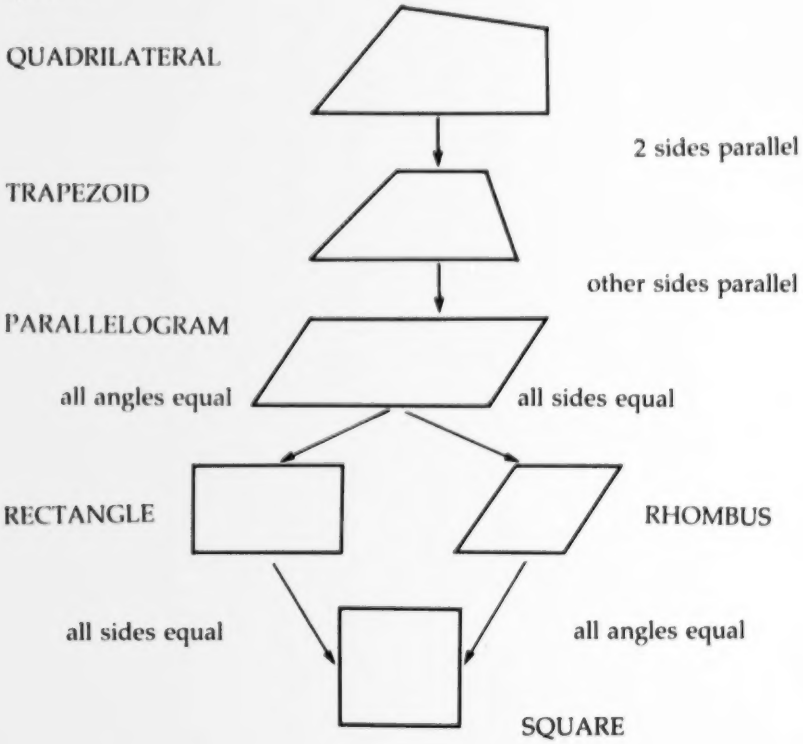

FIGURE 2. Geometric Technical Taxonomy: 4-Sided Figures

Figure 2 shows a category system for 4-sided figures. The most general form of the 4-sided figure is the quadrilateral which requires only that a figure be 4-sided and closed. From the *quadrilateral*, one can produce the *trapezoid* by further requiring that one pair of opposite sides be parallel. To generate a *parallelogram*, then, one need only to require that the remaining pair of opposite sides be parallel. Given the parallelogram, one may impose either one of two further requirements: (1) that all sides be equal or (2) that all angles be ninety degrees. In the first case, one has a *rhombus*, while in the second case one has a *rectangle*. Finally, the *square* can be generated from a rhombus by requiring that all angles be ninety degrees, or from the rectangle by requiring that all sides be equal. This category system for quadrilaterals, unlike the category system for triangles, is hierarchical. Names for general 4-sided figures can apply to more specific 4-sided figures but not vice-versa. For example, a trapezoid is a quadrilateral but a quadrilateral is not a trapezoid, and a parallelogram is a trapezoid but a trapezoid is not a parallelogram. Hence, unlike triangles where each type is equally specific within the system (Figure 1), the category system for quadrilaterals (Figure 2) consists of members that represent different levels of generality or abstraction.

When teaching geometry, teachers are in a position to plan and implement instructional activities for children that are developmental in nature. This is possible if they are sensitive to two sequences of development: (1) Spatial Concepts: Topological → Euclidean & Projective; and (2) Labeling of Shapes: Metaphoric → Proto-euclidean → Technical. Both of these sequences are important to recognize in order to keep learning expectations realistic and to have direction in planning instruction. Teachers can effectively assist children in learning and remembering the names of the different types of three- and four-sided figures by applying these developmental precursors to Euclidean concepts and the logic underlying the technical categories for naming three- and four-sided figures.

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MATHEMATICS *DARES* IN THE OKLAHOMA CITY PUBLIC SCHOOLS

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The history of educational programs bears witness to examples of quick fixes which fixed nothing at any speed. Instances also exist where classroom-proven ideas that were merely collected, coordinated and disseminated quickly improved district learning programs.

This article describes a program of the latter kind. The heart of the program is the use of sequences of Daily Assorted Review Exercise Sets (DARES) developed throughout 1984 by teachers in the Oklahoma City Public Schools. A DARES consists of several questions or problems which are related to previously presented, widely varied course topics (one problem per topic within the set), the set being related by strategic similarity and contrast to other DARES in a day-to-day sequence. An example of a four-day sequence of DARES for arithmetic appears in Figure 1.

Day 1

1. $\begin{array}{r} 231 \\ +457 \\ \hline \end{array}$ 2. $\begin{array}{r} 9763 \\ -4301 \\ \hline \end{array}$ 3. $\begin{array}{r} 237 \\ \times\ 12 \\ \hline \end{array}$ 4. $6\overline{)365}$
5. Complete the boxes: $(6 \times 10^{\square}) + (5 \times 10^{\square}) + 3(10^{\square}) = 6053$

Day 2

1. Add: $974 + 566 =$ 2. Subtract $4936 - 1943 =$ 3. $\begin{array}{r} 278 \\ \times\ 25 \\ \hline \end{array}$ 4. $8\overline{)644}$
5. Round 56381 to the nearest hundred.

Day 3

1. Find the sum: $6142 + 3756 =$ 2. Find the difference: $9408 - 8496 =$ 3. $\begin{array}{r} 593 \\ \times\ 14 \\ \hline \end{array}$ 4. $3\overline{)812}$
5. If a rectangle has length 7 and width 13, find the area.

Day 4

1. $\begin{array}{r} 582 \\ 143 \\ +214 \\ \hline \end{array}$ 2. $\begin{array}{r} 300 \\ -299 \\ \hline \end{array}$ 3. $\begin{array}{r} 196 \\ \times\ 37 \\ \hline \end{array}$ 4. $9\overline{)9815}$
5. Which equals 1? $\frac{1}{2} + \frac{1}{2}$; $\frac{1}{3} + \frac{1}{3}$; $\frac{1}{4} \times \frac{1}{4}$; $\frac{1}{5} - \frac{2}{5}$

Figure 1
The First Four Days of a DARES Sequence for Arithmetic

In Figure 1, the first day's DARES consists of one addition problem, one subtraction problem, one multiplication problem, one division problem, and one non-computational problem. The second day's DARES consists of different examples of the same variety of problems, and so on. Problems gradually become more difficult, and topics shift in and out of the sets as the sequence moves through the year.

Use of the DARES usually involve a procedure similar to the following: following:

1. One day's DARES is given to the students *as a set*, on the blackboard, overhead, ditto sheet, or (optimally) the textbook page.
2. Students work uninterrupted on the DARES for 2-3 minutes while the teacher does uninterrupted housekeeping (attendance, returning graded papers, etc.). The teacher does not help any student until all have had the chance to attempt the DARES *as a set*.
3. The teacher does the problems for the students for review instruction, aloud but at teacher's speed.
4. Papers are collected for an effort grade. Teachers might collect papers prior to step 3 as needed for motivation or credible evaluation and ongoing diagnosis. Students might otherwise exchange papers, etc.
5. Regular class work begins. Once established, the DARES routine is usually finished 5-6 minutes after the beginning of the period.
6. Exams (if used) are cumulative throughout the year, with students informed of this policy at the beginning of the course.

DARES differ from other traditional instructional components in that fairly extensive review is briefly provided each day. Instruction and practice on specific topics is extended over longer periods of time, and questions or problems are presented daily in close proximity to other types of questions or problems. Students are expected daily to demonstrate their knowledge of facts, skills, or concepts.

Much research in educational psychology supports these changes over traditional textbook and classroom expectations. Research findings indicate that most forgetting occurs within 24 hours of initial learning (Horowitz, 1984), and that most of what is learned is forgotten in 24 hours (Buzan, 1983). Also, within a given amount of practice time, or within a set number of exercise problems, numerous brief, widely-spaced practice sessions produce better gains than one or two practice sessions of greater duration (Woodworth & Schlosburg, 1954). Further, short-term retention is enhanced in at least one type of situation by back-and-forth practice on new and old topics (Underwood, 1983), and long-term retention is aided by review (Weaver, 1976). (The term "review," as opposed to "repetition," implies the passage of time between practices on a particular topic.)

Also, turning back in the book to find varied problems is clumsy to

implement and not helpful to achievement or retention (Mora, 1983-1984; Robinson, 1981). Finally, concept formation is facilitated by presentation of related but contrasting ideas side by side on a daily basis (Johnson, 1983).

Obvious benefits of the use of DARES include maintenance of skills for easier learning of more advanced ideas based on these prerequisites, as well as for minimizing reteaching. Serendipity benefits include ongoing diagnosis (especially helpful for mid-term transfer students) and the increased likelihood for some daily success and consequent positive attitudes toward self and subject. Also, teachers whose students move from room to room when classes change can easily direct students to a DARES task at the beginning of the period, which allows for efficient and accurate attendance-taking, etc. Studies clearly demonstrate the supreme importance of the first few minutes of class time for getting students involved with the subject content (Saxon, 1980).

A handful of textbook authors have incorporated DARES characteristics with the exercises which routinely follow each day's discussion of new material. Most notable of these authors is John Saxon, whose *Algebra I* has now been subjected to several scholarly studies and was shown superior in terms of student learning (McBee, 1982; Reed, 1983; Melvin, 1984; Lester, 1983-84). Rhoad, et al. (1984) also incorporated DARES principles into daily exercise sets. Several other school districts, notably the Dallas Independent School District and the Fremont Union High School District in San Jose, have written year-long DARES sequences for each grade or course for mathematics. Teacher/principal acceptance and benefits to students have been happily and inexpensively realized. The Oklahoma City DARES program has been sold to 45 other Oklahoma school districts and to educators and districts in 25 other states.

The development of DARES in Oklahoma City was brief and productive. Four volunteer teachers and the curriculum supervisor for mathematics met for one hour. The process began with the decision to write a 40-day sequence of five-problem DARES for an agreed-on scope and sequence. Each of the five persons was then assigned a type of problem and a problem number for each of the forty sets. Forty sheets of paper were numbered consecutively. The person in charge of question 1 wrote an addition problem on sheet #1, and passed that sheet on to the person in charge of problem 2. In this manner the 40 sheets were passed among the five people. A teacher later copied the sets in condensed form, as illustrated in Figure 1, to reduce the number of pages needing duplication.

The district-level experiences mentioned above and feedback from the author's out-of-district presentations to thousands of teachers and administrators indicate that a program of DARES sequences is quickly perceived to be more than a quick fix. Further development and re-

finement of the descriptions, possible forms, and benefits of DARES remains to be done. Meanwhile, sufficient rationale, evidence, and capability exists for any school district to begin immediately to develop and use DARES sequences, and to argue for inclusion of this structure of student practice in commercially published curriculum materials.

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IMPROVING ARITHMETIC SKILLS

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For years, indeed for decades and generations, elementary school teachers have fretted and worried over their children's inability to learn arithmetic skills. Countless thousands of hours in the elementary grades have been devoted to drill sheets, to flash cards, to colorful manipulatives, to notes to parents — all concerned with Johnny and his arithmetic skills. One can only symphathize with teachers and their frustrations on this point, for what they have taught Johnny and Susie in May often shows no signs of ever having been taught in September. It is almost as if the children had never heard of long division, or how to "carry" in addition. Therefore, the authors decided to conduct a multi-year experiment in the primary grades of the campus school at SUNY-Oswego to determine if a better way of learning arithmetic facts and skills could be found.

This project was heavily influenced by the cognitive theories of Jean Piaget and Jerome Bruner. Notions such as children manipulating and constructing, mathematics as a set of structured relationships, and experience leading from one stage to another were foundational assumptions on which the project was built. One additional catalyst for the project was a statement made by two cognitive psychologists and educators at the University of Rochester, Drs. Arthur Baroody and Herbert Ginsburg. In a conversation with the authors, they asserted that if children were taught to develop a rich network of principles, concepts and relationships, then number facts and arithmetic problem solving would be learned in an easier, more efficient, more economical manner.

This became our thesis: children taught to develop a rich network of principles, concepts and relationships would perform better in standardized arithmetic tests than those taught via isolated drills, memorizations, algorithms. In reality the split was not this sharp. The control groups in the first, second and third grades worked in the traditional textbook manner — following the exercises, drills and algorithms the textbooks recommended and used. The experimental groups did not eschew these traditional methods but did place them second to the development of the network of principles, concepts and relationships. Specifically, two days per week were devoted to working with the children and the teachers in

developing curriculum materials and instructional strategies designed to help the students develop networks of principles, concepts and relationships.

The Project

The project ran for three years in the early 1980's with one control and one experimental group in each of the first, second, and third grades. We began with two first grade classes and continued the project in second and third grade as these classes progressed through school. Thus, year one had two groups, year two had four groups, and year three had six groups. The classes stayed together throughout the experiment since the school had just two classes in each grade. Testing was done on second and third graders, not first graders, in the final year of the campus school's existence. (After 125 years the college closed the school in 1982.) The tests used were the mathematics computation subtests of the Iowa Test of Basic Skills for grades 2 and 3, with the pretests and posttests being different forms of the same battery.

This project had many of the factors one would expect to find in most schools: the beginning group was split homogeneously, not heterogeneously, as the first grade teachers would begin only under those conditions; one control group teacher worked hard to have "her children look good"; some experimental group teachers were not able to integrate the work done on helping children develop a network of principles, concepts and relationships with the traditional class assignments. Thus, while not a true experimental design, the project has results that can be related to many elementary school classrooms.

The experimental groups in both the second and third grades outscored the control groups in the pretest-posttest gains. (The third grade results were achieved with a class which had been together for three years, while the second grade results came from a class which had been together for two years.) In the second grade this gain was dramatic, 8.25

TABLE 1
Second Grade Raw Score Means and Gains

	Experimental Group	*Control Group*
Pretest	14.25	14.86
Posttest	22.50	20.86
Gain	+8.25	+6.00

TABLE 2
Third Grade Raw Score Means and Gains

	Experimental Group	*Control Group*
Pretest	24.17	15.66
Posttest	32.58	23.93
Gain	+8.41	+8.27

to 6.00. In the third grade it was less dramatic, 8.41 to 8.27. This may have been due to the fact that the third graders scored so high on the pretest there was little room for improvement on the posttest.

Many observers came to watch the third grade and its methods of operation. All observers were impressed by the enthusiasm showed by the experimental group of students and by their ability to solve problems not before encountered — i.e., 12 x 15 done mentally as 6 x 30. The use of principles, concepts and relationships became an operational reality with these children. One observer took as her task comparing intellectually similar children in each class in regard to their problem solving abilities. She did this by sitting down, with the children chosen, on an individual basis and asking each child to do a particular arithmetic problem. These problems were chosen from the homework assignments of each group. Hence each child often had a familiar and unfamiliar problem to do. Uniformly the control group children, even the best ones, used rote procedures, and when confronted with a new problem said, "We haven't studied this yet; I don't know how to do it." The experimental group children tried all problems willingly and without prodding, often producing imaginative and different solutions. As the observer said in her report, "The control group will do only that which it has been taught. The experimental group will try anything — and with interest."

The Methodology

The methodology used to help the students develop their network of principles, concepts and relationships was a structural one. That is, they learned the structure of the operation they were doing, not merely its algorithmic form. From the first grade on the children learned that $2 + 3$ could be looked at as $2 + (2 + 1)$, and hence the answer of 5 would be twice the original number plus one. This in turn, and in the second grade, led to a discussion of odd and even numbers. Finally in the third grade a working, but not formal, definition of odd numbers was developed where an odd number was any number doubled with one added or subtracted. In the fourth grade $2K \pm 1$ would have emerged as a formal definition.

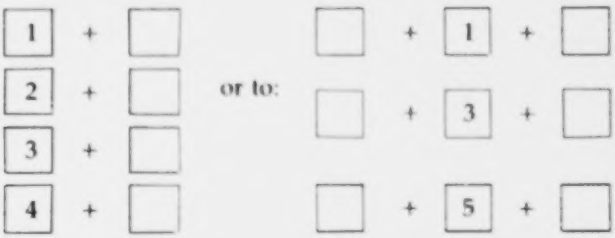
Followers of Bruner will see a spiral curriculum approach at work here, while followers of Piaget should see a constructivist approach. Those who read Morris Kline's *Why Johnny Can't Add*, criticizing the modern math of the 1960's, should also see his emphasis on the formal emerging from the informal, or the intuitive preceding the logical. All this was intentional, and consistent throughout the curriculum. In fact, the structural approach used emphasized two types of structures: (1) the student's own constructive structures, those each individual would build for him or herself and (2) the structures of the discipline, those which mathematics itself had evolved over centuries. This combination of the logical with the intuitive, the formal with the informal formed the heart of the curriculum.

The materials used were those taken from commercial texts, but transformed according to the dual thrusts of the student's own structures and those structures found in the discipline of mathematics. An example would be that of a usual presentation, in grade one, of 2 + 3. Often this was done as:

Instead we would ask the children to look at the following configuration and describe, or write the combination (patterns) they saw:

Here it is possible to see not only 4 + 1 and 2 + 1 + 2, it is also possible to "see" or to construct 1 + 3 + 1, as the diagram on the right shows.

Such seeing, or working with patterns, is, of course, the heart or core of mathematics. From the patterns constructed with five (5) one can, in later months, move to:



Both of these patterns lead to interesting configurations involving fractions, negative numbers, and exponents. For example, $1^2 + 2^1 + 3^0$ is a pretty pattern which almost, but not quite, equals five. Trying to find an interesting and intriguing pattern of advanced sophistication equal to five filled many hours of third grade classroom and homework time.

Another exercise in patterning came from grids and from "arrow games" played with grids. A simple grid (base 10) would be:

20		22		24		26		28	
	11		13		15		17		19
0	1	2	3	4	5	6	7	8	9

An "arrow game" one could play on this grid would be:

2 → → = [4] 3 ↑ ↑ = [23]

Varieties would be:

5 ↑ ↑ → ↗ = [ ] 7 ↗ ↖ = [ ]

Obviously these could, and did, become quite sophisticated, especially with the introduction of negative numbers and bases other than ten.

Other devices were also used, such as multiplying 16x5 as 50 + 30, or as 8x10, and subtracting 49 from 101 as 50 from 102. But in all devices used, the principles of personal construction and mathematical structure were predominant and paradigmatic. They were guided forces of curriculum development. The students in the experimental groups did well, we believe, because in large part they were able to develop intuitively and operationally, a rich network of principles, concepts and relationships. In short, they developed a sense of mathematics and what it is all about. More students would do better on more mathematics tests if such a sense were developed.

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SYNTHESIZING RESEARCH-BASED INSTRUCTIONAL STRATEGIES FOR EFFECTIVE TEACHING

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Reports issued by national commissions (National Commission on Excellence in Education, 1983; National Science Board Commission on Precollege Education in Mathematics, Science and Technology, 1983) and prominent researchers (Boyer, 1983; Goodlad, 1984) have indicated that our nation's schools are in great need of improvement. Their recommendations emphasize the need to upgrade the quality of curricular and instructional programs, particularly those in science and mathematics. At the same time, researchers have been reporting the results of their efforts to determine what makes some schools, some classrooms and some instructional practices effective, while other schools, classrooms and practices are not so effective. (See, for example, Edmonds, 1978; Goodlad, 1984; Bloom, 1984).

As a result of the public demand for excellence and the concurrent release of the research results, many school districts have conducted inservice programs that deal with effective teaching strategies, time on task, direct instruction and basic skills. Unfortunately, the ultimate effectiveness of many such programs may suffer from their having a too narrow focus. Inservice programs that deal only with effective teaching strategies can fail to focus on the object of the teaching. Time on task programs are short-sighted if they fail to inquire into the nature and quality of the tasks. Direct instruction strategies may be quite appropriate at certain times, but are highly unlikely to be appropriate for all kinds of curricular objectives and instructional situations. Programs that focus on basic skills ignore the need for students to deal with higher-order thinking.

Teaching is a complex act and good teachers need to focus on a wide array of curricular and instructional variables in their improvement efforts. Johnson (1986) has called on supervisors to design inservice programs that focus on classroom management methods that will help teachers learn how to efficiently incorporate diagnostic procedures, begin the class period and get students on task, and obtain and use feedback from students. Teachers need to use instructional models that incorporate strategies and resources which can address a number of the aforementioned critical concerns and ideas. They need to implement gestalt, rather than single-focus, models in their classrooms. The remain-

der of this article examines one such model.

Instructional Model for Flexible Grouping

The author, in cooperation with more than 100 elementary school teachers, developed and refined an instructional model for elementary school mathematics that is built on the concepts of diagnostic-prescriptive teaching and mastery learning. Diagnostic-prescriptive teaching assumes that expectations (objectives) can be rather clearly delineated, that appropriate instruments or procedures can be developed to assess student achievement of those expectations, and that there are resources available to help those students who do not have the competence specified in the expectations (Nicely, 1977). The concept of mastery learning is based on Bloom's 1968 assertion that "most students can master what we have to teach them, and it is the task of instruction to find the means which will enable our students to master the subject under consideration. Our basic task is to determine what we mean by mastery of the subject and to search for the methods and materials which will enable the largest proportion of our students to attain such mastery."

Essentially, the following model is a plan for developing and implementing instructional units. Each unit is based on a set of objectives, some of which students must master, and others of which are optional at the discretion of each teacher and/or the local curriculum committee. Each unit includes diagnostic procedures which are designed to help teachers assess student achievement of the essential objectives and a variety of instructional strategies and materials related to these objectives. A plan for classroom organization and *management that includes within-class flexible grouping* is necessary for the successful implementation of each unit.

The model does not require continuous progress across grade levels or units, but rather allows students to work on optional or enrichment objectives and activities once they have demonstrated mastery of the locally determined essential curriculum. Experience has shown that the model does not require additional personnel, but is quite manageable for one teacher with about thirty students (Nicely & Shepler, 1971). It is designed to increase the amount and quality of instructional attention given to individual students by grouping and regrouping them based on the results of locally developed diagnostic procedures.

It can be adapted to the resources that are available in any given classroom, whether that classroom contains only minimal instructional resources, or whether it contains a relatively rich materials environment including multi-media materials and easy access to computers. Provision is made for most students to attain mastery of the locally determined essential curriculum, and teachers can systematically provide reinforcement of this basic curriculum. A variety of ways for students to apply the basic content can be provided, and students are likely to be involved in making some of the decisions. Optional objectives are avail-

able to students who have mastered the essential curriculum.

The instructional flowchart in Figure 1 shows the decision-making and

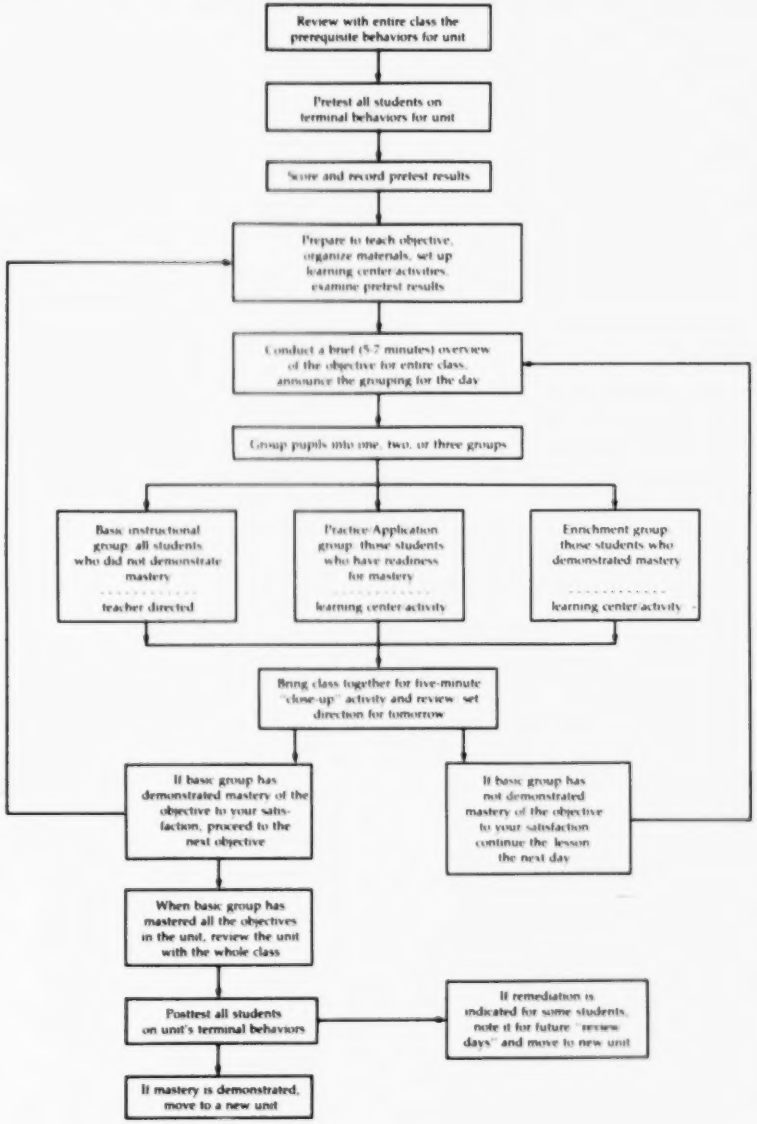
Review with entire class the prerequisite behaviors for unit
Pretest all students on terminal behaviors for unit
Score and record pretest results
Prepare to teach objective, organize materials, set up learning center activities, examine pretest results
Conduct a brief (5-7 minutes) overview of the objective for entire class, announce the grouping for the day
Group pupils into one, two, or three groups
Basic instructional group: all students who did not demonstrate mastery
teacher directed
Practice/Application group: those students who have readiness for mastery
learning center activity
Enrichment group: those students who demonstrated mastery
learning center activity
Bring class together for five-minute "close-up" activity and review; set direction for tomorrow
If basic group has demonstrated mastery of the objective to your satisfaction, proceed to the next objective
If basic group has not demonstrated mastery of the objective to your satisfaction continue the lesson the next day
When basic group has mastered all the objectives in the unit, review the unit with the whole class
Posttest all students on unit's terminal behaviors
If remediation is indicated for some students, note it for future "review days" and move to new unit
If mastery is demonstrated, move to a new unit

Figure 1
INSTRUCTIONAL FLOWCHART FOR FLEXIBLE GROUPING

instructional management processes that are used by teachers for instruction within a curriculum unit. The flowchart contains many of the critical elements for effective instruction that have been identified by Bloom (1984) as well as those that have been included in Hunter's instructional model. Before beginning instruction in the unit, the teacher provides "initial enhancement of cognitive prerequisites" (Bloom, 1984) by reviewing with the entire class the prerequisite behaviors for the unit. At the end of that review session, the teacher pretests all students on the terminal objectives for the unit and then scores and records pretest results. Utilizing a simple form (see Nicely, 1977), a teacher can easily record and later identify which students have mastery, readiness for mastery, or no knowledge regarding each objective in the unit.

When preparing to teach any of the unit's objectives, the teacher will organize the materials, set up learning centers and activities, and look at the student performance chart to determine instructional groupings for the day. At the beginning of instruction each day, the teacher conducts a brief overview of the objective for the entire class (Hunter's anticipatory set) and announces the instructional groupings. Those students who have demonstrated mastery of the objective on the pretest can work at an activity designed to reinforce and/or extend their knowledge about that objective or to provide an introduction to another objective which is outside of the essential curriculum but which might be valued by the teacher. The students who have readiness for mastery will usually work at structured activities which are designed to give them instruction and then do appropriate practice to help them overcome their demonstrated weakness. The third group (the basic instructional group) is composed of those students who did not demonstrate mastery and need direct instruction and supervision from the teacher. (This group typically receives instruction which is most like that which all students would receive if the teacher knew nothing about their competence on the objective prior to instruction.) Throughout the class period, the teacher moves from group to group providing instruction, feedback, guidance and assistance as necessary. Near the end of the class period, the teacher brings the students together for a 5-minute close-up activity and review. This closure activity also enables the teacher to specify the direction for the next day since she/he has had an opportunity to collect formative data and monitor student progress.

If the basic group has demonstrated mastery of the objective to the teacher's satisfaction, the next day's instruction will focus on the next objective. However, if the basic group (or some subset of it) has not demonstrated mastery of the objective to the teacher's satisfaction, the lesson may be continued the next day, with a possible regrouping of students. This decision-making and instructional management cycle repeats until the students have mastered all of the objectives in the unit. At that time, the teacher reviews the entire unit with the entire class and

posttests the students on the unit's terminal behaviors. From a management standpoint, most teachers have decided that if remediation is indicated for only two or three students, it will be noted for future "review days" and the class will move to a new unit. If, however, a large number of students still have not mastered an objective, additional time may be spent for remediation.

The most recent modification to the flexible grouping plan was reported at the 1985 PCTM conference. During their slide and transparency-augmented presentation, Nicely and Stonebraker (1985) illustrated how attention could be given to problem-solving within the elementary school mathematics program while at the same time using diagnostic-prescriptive teaching strategies along with mastery learning, direct instruction, curricular enrichment, and flexible instructional grouping. Figure 2 shows how the time that was allotted for mathematics instruction in a sixth grade class was utilized to integrate problem solving with the flexible grouping/mastery learning program.

TOTAL TIME IN PERIOD	SUBDIVISIONS OF TOTAL TIME
45 minutes	10-15 minutes: problem solving activity with the entire group 5 minutes: introduction to whole group 20-25 minutes: small groups 5 minutes: wrap-up with whole group

Figure 2
TIME ALLOTMENT FOR ONE CLASS PERIOD

The first 10-15 minutes of each class period were used for whole-group problem solving activities utilizing the *Mathematical Problem Solving* materials produced by Charles and Mason. The remaining 30-35 minutes were devoted to the school district's regular mathematics program (as directed by the district's planned course for mathematics at grade six) utilizing the instructional flowchart for flexible grouping that was illustrated in Figure 1.

In order to successfully implement such an instructional management plan, teachers must be well-organized. Figure 3 illustrates the actual instructional resources guide that was developed by Stonebraker (principal) and McIlvaine (sixth grade teacher). The instructional resources guide serves as the "action document" for the teacher and illustrates how attention is paid to issues such as congruence among the written, taught, and tested curricula, basic skills, problem solving, differentiated instruction for students, and the identification, acquisition and utilization of locally available resources for instruction in each objective.

Each objective in the mathematics curriculum can be assigned a unique five-digit number. For example, the 60401 (see upper right-hand corner of Figure 3) indicates that this objective is taught in the 6th grade, fourth

Unit Name Division of Whole Numbers
Unit Number 604

Objective Number 60401

OBJECTIVES	INSTRUCTIONAL MATERIALS/RESOURCES/PROCEDURES		
Students will choose and execute a solution strategy to solve a *process* problem.	Whole group activity. Use *Mathematical Problem Solving*, Teachers Manual (Charles and Mason), pages 46-47.		
Students will divide a number 99,999 or less by a number 9 or less. $6\overline{)90{,}300}$	BASIC (0/4 or 1/4) Day 1 1. Teacher Directed — Have students turn their paper sideways to copy problems. Use *Macmillan*, Book 6, page 99, problems 1-5. Do problems together. 2. Classwork/Homework — Remainder of page 99. Day 2 1. Teacher Directed — Use *Scott Foresman* worksheet #P26 2. Homework — *Macmillan*, page 101.	PRACTICE/APPLICATION (2/4) Day 1 1. Independent — Have students use *D. C. Heath*, Basic Worksheet for 74-75. After completing the first five problems, check with answer key. 2. Homework — Complete worksheet. Day 2 1. Small Groups — Use gameboard "Rally with Remainders — Level 3" 2. Homework — *Macmillan*, page 101.	ENRICHMENT (3/4 or 4/4) Day 1 1. Small Group Work — Have students work in pairs (or triples) to complete puzzle page. Use *D. C. Heath*, Enrichment Worksheet for 78-79 and chalkboards. 2. Homework —Complete worksheet. 3. *Mystery Mansion* learning center. Day 2 1. *Mystery Mansion* learning center.

Figure 3
INSTRUCTIONAL RESOURCES GUIDE

unit, and is the first objective in that unit. The problem-solving objective is specified and the appropriate pages in the Charles and Mason resource are identified. The remainder of the instructional resources guide is organized so that there is a high probability of congruence among the written, taught and tested curricula. The objective from the planned course guide is specified, along with a sample test item. The instructional resources (the taught curriculum) are listed for the three (potentially) different groups of students. All of the materials are related to the specified objective or an extension/application of that objective to some other area.

Implications for Instructional Leadership

The instructional model for flexible grouping that was first designed seventeen years ago has evolved as the knowledge and experience of the original designers have increased and as new problems have arisen to be solved. The model attends to many current and important curricular and instructional concerns including congruence among the written, tested and taught curricula, basic skills and problem solving in mathematics, enrichment activities, mastery learning, diagnostic-prescriptive teaching, direct instruction, individual differences, productive time-on-task, and effective teaching strategies. The model will continue to evolve and will incorporate microcomputers and associated software — both for instruction and management — as these resources become more accessible to teachers and students (Nicely, 1984).

This mastery learning-based instructional model is not being proposed as the answer to all instructional problems that exist, but rather as an illustration of one way to deal with the complex and interacting variables that face classroom teachers. Other models, or adaptations of this model, may be needed in order to deal effectively with different grade levels or environmental conditions. Mathematics teachers and supervisors should be designing, developing and using instructional models that can be successfully implemented in the classroom. These instructional models should incorporate those elements which have been gleened from sound research studies and successful, practical efforts and should avoid simplistic solutions to complex instructional programs.

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1886-1986: THE STATUE OF LIBERTY AND SOME MATH MAGIC

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Since 1986 is the year in which we shall observe the one hundredth anniversary of the dedication of the Statue of Liberty, the numbers 1986, 86, 1886, and 100 can be used as goals for "magic" squares, stars and circles, a "magic" cube, and mathematical limericks. The anniversary problems and situations described in this article have been shared with teachers in workshops conducted under the auspices of the Pennsylvania Department of Education and the Pennsylvania Council of Teachers of Mathematics. They have also been presented to secondary school students, many of whom found the ideas to be stimulating.

These unexpected and challenging forms can be used to motivate students to apply arithmetic, algebra, and geometry in directions which may be new to most of them. A good reference for readers who wish to pursue these and similar ideas is *Magic Squares and Cubes* by W. S. Andrews, Dover Publications, Inc., New York. The magic cube and magic sphere in this book can provide three dimensional insights and explorations.

The limerick shows one relationship that can be established between language and mathematics. Of special interest is the seven-pointed star which is achieved (approximately) by using the apothem of the inscribed hexagon as the chord for the inscribed heptagon.

MAGIC SQUARES

1886: Horizontally, Vertically, Diagonally

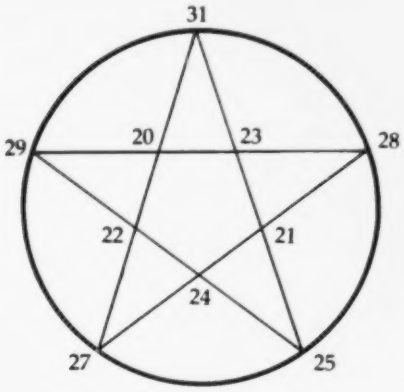
404	530	521	431
503	449	458	476
467	485	494	440
512	422	413	539

1986: Horizontally, Vertically, Diagonally

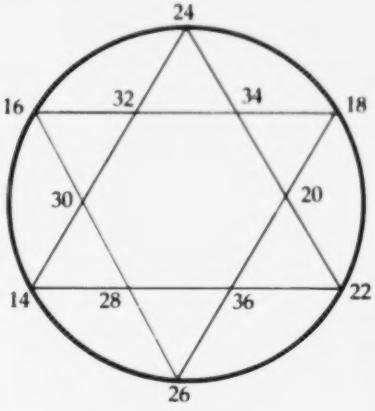
504	491	490	501
493	498	499	496
497	494	495	500
492	503	502	489

STARS AND CIRCLES

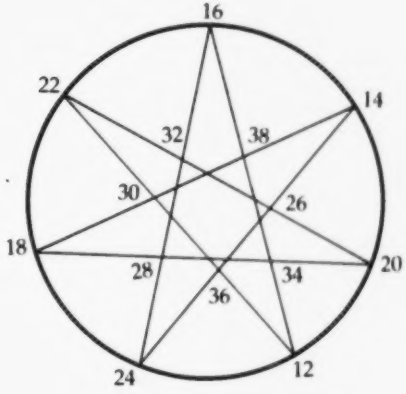
100: Five-pointed star.
Sum of 4 points on a line = 100.



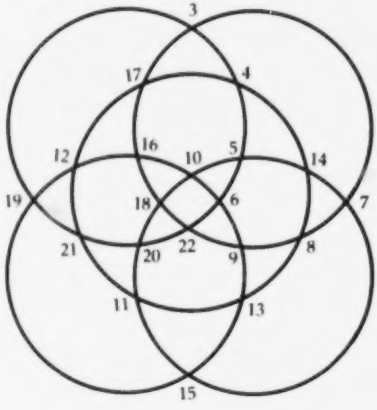
100: Six-pointed star.
Sum of 4 points on a line = 100.



100: Seven-pointed star.
Sum of 4 points on a line = 100.



100: Intersecting circles. Sum of 8 points on each circle = 100.



MAGIC CUBE

This 3 x 3 x 3 cube totals 1986 horizontally, vertically, and diagonally *through the cube!*

Top layer

658	674	654
672	649	665
656	663	667

Middle layer

671	651	664
655	662	669
660	673	653

Bottom layer

657	661	668
659	675	652
670	650	666

LIMERICK

$$\frac{12 + 144 + 20 + 10\sqrt{4}}{7} + 5(11) + 3 = 86$$

The above to be read:
A dozen, a gross and a score,
Plus ten times the square root of four,
Divided by seven,
Plus five times eleven,
Plus three; *eighty-six* is our score.

K-8

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WHOLE-BRAIN THINKING IN THE MATHEMATICS CLASSROOM

by
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Research dating as far back as 1910 indicates that the two sides of the brain perform different functions. The fact that our brain possesses two hemispheres with two diverse cognitive styles is no longer in question. In most people the left hemisphere processes information in a sequential, linear and analytical manner, playing a primary role in such activities as reading, writing, arithmetic and algebra. The right hemisphere tends to specialize in nonverbal skills, spatial visualization and in making sensory discriminations. When one engages in an activity dominated by one hemisphere, the other side of the brain does *not* lie inactive. There is always a degree of communication between the two through a bundle of nerve fibers known as the corpus callosum. It is widely believed that communication between the hemispheres, that is between the two modes of knowing, is a vital process, a necessary condition to creativity.

In 1981, neurosurgeon Roger Sperry won the Nobel Prize for this proof of the split-brain theory, which says that our lives are greatly influenced by our habit of using one side of the brain more than the other. Innovative companies such as IBM, ARCO, Kodak, Sperry Rand and Dow are conducting dramatically successful seminars and workshops on whole-brain thinking. According to experts in the field of management consulting, one of the secrets to peak job performance and satisfaction is learning to use both sides of the brain. Learning to recognize the thinking styles of others and to alter or adapt to them is thought to be an essential tool to effective communication. It is time for educators to take serious notice of this research and its implications for instruction.

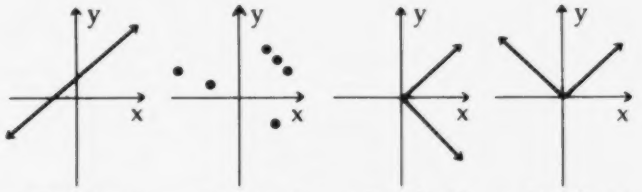
Many of you are familiar with students' complaints such as, "I was always good in algebra," — mostly left hemispheric oriented — "but I can't understand geometry!" — predominantly right hemispheric oriented. Or, "Geometry was fun, but algebra is boring." There is a definite duality here — students who do well in algebra, but can't cope with geometry, or the other way around. Of course there are those who do well in both — are they dual hemispheric? In higher grades, many students encounter more and more difficulties in mathematics. Could it be because higher level mathematics requires strong input from both hemispheres and a higher degree of communication between them?

From observations and testing in the author's classes, it appears that taking into consideration hemispheric theories when teaching math-

ematics might be a very important factor in facilitating students' successful learning of mathematics. A practical strategy is to determine students' perceived hemispheric specialization and the hemispheric nature of the material and then to develop techniques to teach the topics to both hemispheres. A Brain Preference Indicator (self) Test that correlates with EEG dominance tests can be found in Chapter 2 of *Whole-Brain Thinking* by Wonder and Donovan (1984).

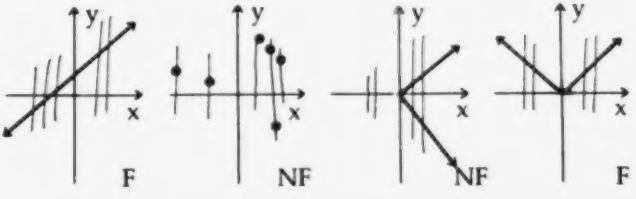
Is there a "right way" to solve a problem correctly? Depending on the hemispheric strength of an individual there can be many "right ways" to arrive at the correct solution. The following examples show how differently three students responded to the same examination questions. The material was taught by the same teacher at the same time. There was no "best way" emphasized, rather several methods were demonstrated, stressing freedom of choice. It is clear from the way answers are worked out and given that each student perceived and learned in a different way. The method chosen by each student seems to be very much in line with the author's perception of each student's hemispheric specialization. (Through testing and observations, Student "A" appeared to be strongly left hemispheric, Student "B" to be strongly right hemispheric and Student "C" to be dual hemispheric, where neither hemisphere dominates.)

Question: Use the vertical line test to determine if the following graphs represent functions or not. (Answer: #1 & 4 are functions. Note: If no vertical line can be found that intersects the graph in more than one point, then the graph is said to be a function.)



Graph #1 Graph #2 Graph #3 Graph #4

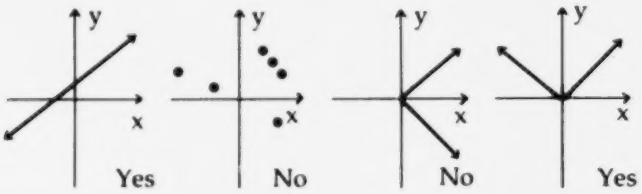
Response of Student "A"



F NF NF F

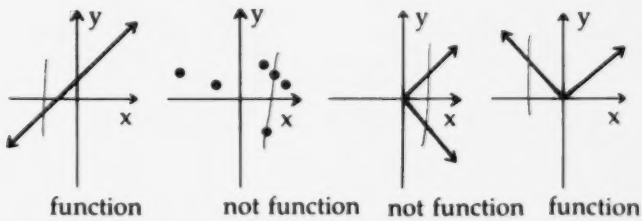
Strongly left hemispheric student "A," being methodical and sequential, drew several vertical lines on each graph to help her decide. She had little spatial input from her right hemisphere and apparently could not image those lines along the graph. Rule application is said to be also very characteristic of left hemispheric processing and she is clearly doing that in a very methodical fashion. She identifies functions with an "F" and not functions with an "NF" — very logical symbolism, again believed to be a very strongly left hemispheric trait.

Response of Student "B"



Strongly right hemispheric Student "B" did not need to draw any lines to help her determine whether there were any that intersect in more than one point. She apparently could clearly picture all of them in her mind (her right hemisphere) without the lines, she knew the right answer instinctively, and she trusted her intuition and answered the actual question with "yes" or "no."

Response of Student "C"



Dual hemispheric Student "C" also saw the correct answer with input from her right hemisphere, but her logical, methodical left hemisphere compelled her to check the solution with a few strategically placed lines on each graph. She gave her answer directly by naming each as "function" or "not function." One might ask, did input from her left hemisphere prevent her from answering with the abstract "yes" or "no" and at the same time did input from her right hemisphere keep her from using symbols?

Question: If $f(x) = 2x - 3$, find $f(-1)$, $f(0)$ and $f(2)$ (Answer: $f(-1) = -5$, $f(0) = -3$ and $f(2) = 1$. Note: substitute -1, 0, and 3 respectively for x.)

Response of Student "A"

$f(-1) = 2(-1) - 3$	$f(0) = 2(0) - 3$	$f(2) = 2(2) - 3$
$f(-1) = -2 - 3$	$f(0) = 0 - 3$	$f(2) = 4 - 3$
$f(-1) = -5$	$f(0) = -3$	$f(2) = 1$

Response of Student "B"

$f(-1) = -5$ $f(0) = -3$ $f(2) = 1$

Response of Student "C"

$2(-1) - 3$

$-2 - 3$ $\{-5, -3, 1\}$

Examine these responses from the three students and compare them with those given in the first example. Note how a pattern seems to emerge from just two instances per student. You should start observing your own students to see if you can identify hemispheric bias. Then you can alter your presentation modes to provide instructional alternatives based on individual hemispheric differences.

The challenge for mathematics teachers is to make the best of available research findings and attempt to apply it in the classroom. Mathematics is not one dimensional, but a combination of science and art. Intuition is just as crucial to the evolution of knowledge in this field as orderly, formal and logical thinking.

We need to make deliberate efforts to stimulate both sides of the brain and draw on both modes of knowing in order to promote better understanding and improve learning. We need to develop and improve techniques that allow the teaching of the same material to both hemispheres and at the same time encourage the development of the nondominant one. We should also examine and determine our own hemispheric bias and make a conscious effort to strengthen the performance of our weaker one. The benefits can be great both in professional and in personal terms.

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A MODEL AS A STARTING POINT FOR TEACHING PROBLEM SOLVING

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Consensus that problem solving should be emphasized more in the schools has been well documented in the past few years. Educators and lay people alike have been in agreement that this is a need of high priority (National Council of Teachers of Mathematics, 1981). In spite of this, however, much confusion exists about its nature and implementation. A good problem solving model can address several aspects of this problem. It can help define what is meant, thereby helping teachers to understand the place, purpose, and value of higher-level problem solving. It can also provide teachers with guidelines as to how to help students proceed when the method of solution is not immediately obvious.

Defining what is meant by solving "problems" is the first hurdle to be overcome. Opinions range all the way from finding the sums of a group of numbers on a worksheet to solving a problem to which no method of solution can be thought of. Because of such widely divergent definitions, some teachers may believe they are already teaching problem solving when actually they are not, according to those emphasizing the higher order thinking skills.

For the purposes of this paper, "problems" addressable by the model will include only those for which at least some thinking is required in order to know how to proceed. Examples would include word problems such as "how many hair curlers could you get into a refrigerator?" (unfamiliar problem) and "Jack has 5 dollars and Jill has 4 dollars. How much does Sue have?" Note that the last example is "familiar" in that it suggests a procedure. Also note that it does not have a solution, a fact that in no way prevents it from being considered to be "familiar."

Other factors make implementing problem solving difficult. Popular support for the back-to-basics, testing, and accountability movements, with their emphasis on drill and practice and "cut and dried" procedures and outcomes, for example, may be contradictory to the divergent thinking emphasis in problem solving. Use of a model may therefore be helpful in convincing these proponents of the need for this "other" emphasis in the curriculum by helping them to see the value of and better understand the processes involved.

A third reason for concern is the extent to which thinking skills are now being incorporated into the schools. Research relating to classroom practices has produced some discouraging conclusions. The most current mathematics assessment (National Assessment of Educational Progress,

1983), for example, indicates that school children have continued to perform poorly in higher-level problem solving. Other studies have shown that many schools and teachers do not emphasize problem solving and do not provide a climate conducive to problem solving activity.

Merely acknowledging weaknesses in problem solving instruction will not, in itself, help teachers to do a better job of implementing problem solving. A number of educators have suggested that teacher training, preservice and inservice, may be the most promising means to making problem solving more of a reality in the schools. Some have advocated content courses in problem solving so that teachers themselves would better understand the nature and importance of problem solving. Others have suggested methods courses which could assist teachers in improving their delivery of problem solving instruction. Some feel that both should be done. There appears to be even less consensus over what teachers should be learning in these courses, and definitions of that which constitutes problem solving vary. Regardless of personal view, a model which defines and suggests logical procedures would be helpful.

A Model as a Starting Point

There has been a proliferation of problem solving models, each with its own rationale or philosophy. The following model is being proposed because of two basic reasons: (1) it is very simple and (2) it can be used to address some of the major shortcomings and/or misunderstandings of past practices. This makes it ideal for use in helping teachers to become more knowledgeable with the problem solving process themselves as well as to learn how to more effectively convey this knowledge to students. While a listing of steps does not guarantee success in solving a problem, the steps and their order do suggest a logical procedure to follow when a person is at a loss as to how to start. They will also help students from prematurely making decisions which would lead to faulty solutions. With teacher help, students can learn to be successful in working problems that do not have obvious solutions. The Problem Solving Model (PSM) of Troutman and Lichtenberg (1982, p. 266) is outlined in Figure 1.

The most obvious feature of the PSM is the division of problems into two different types, familiar and unusual (unfamiliar). Familiar problems might include the use of the textbook study questions and teacher-made review sheets which are so much in evidence in school practice. They might also include certain kinds of teacher questioning where the teacher is primarily concerned with testing previously learned material. In that familiar problems are frequently encountered in both school and non-school settings, this type of problem should be represented in the school curriculum. Another reason for including familiar problems is their relationship to unfamiliar problems. The last step of solving an unfamiliar problem is identical to the first three steps of solving a familiar problem.

Part 1: familiar problems	Part 2: unusual problems
1. Select established procedures.	1. Become familiar with the problem.
2. Carry out procedures. and find solutions.	2. Collect information related to the problem.
3. Evaluate the solutions.	3. Devise strategies for solving the problem. Evaluate the strategies.
	4. Select a strategy and carry it out to find solutions. Evaluate the solutions.

Figure 1
Problem Solving Model of Troutman and Lichtenberg

Unfamiliar problems include the kinds of problems for which students have not had any prior experience. Such questions as "how much sand will fill a room?" or "what is the size of your brain?" would be unfamiliar problems to many students.

Though the use of unfamiliar problems is not much in evidence in most schools, this type of problem is possibly the most conducive to the development of mathematical thinking skills and the most compatible with the scientific method. It may come the closest to preparing students to successfully cope with the new and different problems that will face them in the future. They represent more the "application learning" associated with the "experimental mind" than do familiar problems based more on "acquisition learning" (fact learning) (Brown, 1968). In that unfamiliar problems are often novel and not directly related to the acquisition learning emphasis of existing curricula, teachers may sometimes think of these problems as silly or trivial. The "strangeness" of these problems is precisely that which makes them good unfamiliar problems. Some of the possible benefits from these problems may include (1) learning strategies for solving future problems (turning what would be unfamiliar problems into familiar problems), (2) encouraging creativity, (3) developing individual self reliance in working with the unknown, (4) developing skill in applying knowledge, and (5) integrating knowledge.

The PSM is useful in solving both familiar and unfamiliar problems in that it also indicates a sequence of procedures which should be followed to logically solve any type of problem which is not superficial. Often the hardest part of a problem is knowing what to do. The Model can be used to help overcome this obstacle. Though the steps and rationale which will now be discussed may seem simplistic and "common-sensical," many

student difficulties are traceable to failure to heed the guidelines. The model should be taken seriously.

Solving Familiar Problems

The first step in solving a familiar problem is to select a strategy. Students may assume that there is only one way to solve problems. They need to be made aware of at least two other possibilities: (1) there may be several correct ways to work a problem or (2) the problem may not even be solvable. Many times there are several correct ways to approach a problem. The best approach may simply be the one which is easiest, fastest, most accurate, or most appropriate based on some other logical criterion.

In the example given earlier (Jack has 5 dollars and Jill has 4 dollars. How much does Sue have?), the student should select the most appropriate strategy from among several possibilities. Some of these might include: add the numbers, subtract the numbers, divide the numbers, or get more information. In this case, getting more information would be the appropriate one since the question has no relationship to the information given. Deciding on and acting on a strategy before considering the alternatives would likely lead to a false conclusion. This example points out the dangers of other approaches such as looking for key words such as "and" and assuming it would mean that addition is called for.

The second step is carrying out procedures and finding solutions. This could be thought of as the "doing" part of the problem and the part that most students seem to instinctively know that they must do. Here the calculations are carried out and the answer is determined. To many students this step *IS* the problem. They should be cautioned about the dangers of proceeding directly to this step (step 2) before selecting an appropriate strategy (step 1). Students could (and often do) get an incorrect answer by performing the wrong computations.

Step 3, evaluating the solutions, is frequently overlooked by students. Once a solution is arrived at, students may have a tendency to want to "get on with it" and do something else (like solve another assigned problem). Many ridiculous solutions might be self corrected by the students themselves if they took the time to think about their results and their meaning. This is not to mention the deeper level of understanding they might gain about the problem and its meaning from evaluating their results. If an incorrect solution is obtained, its unreasonableness might be detected and corrective action taken. In the example given, the lack of relationship between the information given and the question asked might become obvious even at this final phase.

Possibly the greatest hindrance to students implementing step 3 is time pressure. Both teachers and students should be keenly aware that good and successful problem solving experiences will require adequate time and concentration.

Solving Unfamiliar Problems

The PSM reveals that unfamiliar problems are not solved any differently than familiar problems except that one must start farther back in the problem solving sequence. Put one way, this means that the first step in solving an unfamiliar problem is to make a familiar problem out of it. Put another way, steps 1-3 of familiar problems are the last step (step 4) of unfamiliar problems. All that was applicable to the solving of familiar problems is also applicable to unfamiliar problems. Unfamiliar problems, however, make use of three additional steps which are not necessary in familiar problems. The purpose of these steps is to generate appropriate strategies for solving the problem. As in the case of familiar problems, teachers and students alike should recognize that these "extra" steps take time and effort. Failure to consider any of them could lead to a misunderstanding of the problem and incorrect solutions.

The first step in solving a problem where there are no known solutions is to become familiar with the problem. That which first appears to be the problem may prove to be otherwise under closer examination. Clinical psychologists, for example, are keenly aware that human beings may report one problem but that one must often "read between the lines" to find the "real" one. The "real" problem may be completely different in nature from the initially conceived problem. Addressing the "real" problem has the greatest potential for helping the patient. The same analogy is true in science and mathematics. Most teachers have had experiences with students who did a beautiful job of solving the wrong problem. A student's conception of what the problem is asking for may change after adequate time and effort has been given to determining precisely what the problem is.

Another example will help clarify the usefulness of observing this first step: "how many hairs are on your head?" The first step is to make a distinction between the stated problem and the "real," underlying problem. The stated problem suggests some type of counting process. The "real" problem centers around the difficulty of keeping track of that many hairs. Once this is identified as the central issue, it is much easier to think of ways to tackle the problem. This helps steer thinking in a direction most likely to be productive.

After defining the problem, the problem solver may still not know how to proceed. He/she would logically move to step 2, collecting information related to the problem. The problem solver should concentrate on thinking about the knowledge or information that might have a bearing on the problem. Using the "hair" example again, students can often think of parallel situations involving sets of very large numbers. One student may have estimated the blades of grass in a field for a science class assignment. Another may remember how red and white blood counts are estimated from small samples. Once students see these kinds of connections, the appropriate strategy/strategies become evident.

One of the greatest threats to circumventing this process is prematurely deciding on a strategy or strategies before learning pertinent information that may have an important bearing on which strategies might be appropriate. The problem solver should have the capacity to reserve judgment until the appropriate time.

Step 3 is devising strategies. It is best that more than one be generated. Using the clinical psychological example again, a person who recognizes many options to solving a personal problem may feel quite comfortable about solving it. A person perceiving only one option may feel threatened, in that no backups are perceived in case the only option available fails. The person who sees no options may become desperate if the problem is great enough and may require outside assistance. Frustration in solving unusual mathematics and science problems can sometimes be similarly traced to difficulty in generating strategies for solving the problem (getting options).

In the "hair" problem, there are many details to be worked out. Typical questions might include: if the number of hairs is to be estimated from a small sample, how large should the sample be; what kinds of measurements should be made; how should the area of the scalp be measured; and must every hair in the sample be counted? This makes for excellent class discussion where all can contribute to the brainstorming process and where all ideas are valued without being prejudged.

The devising of strategies (step 3) should not be confused with the selection of strategies (step 4). Step 3 is analogous to "brainstorming." Prematurely selecting a strategy before giving the proper attention to strategy generation may result in more appropriate strategies being overlooked.

After all this preliminary work has been done (steps 1, 2, and 3), the student is now in a position to select an appropriate strategy, solve the problem, and evaluate it (step 4). Students may be expected to be most successful with unfamiliar problems when they avoid the following temptations: (1) skip steps that may have a bearing on the solution, (2) ignore the order of the steps, (3) prematurely select a procedure, or (4) rush through the problem too quickly. They should be instructed that successful problem solving takes time and effort and be given some of the reasons why it is worth it.

This paper has advocated the problem solving model of Troutman and Lichtenberg (1982) as a guide to teaching the nature of problem solving and procedures for solving problems. The model can help teachers to better understand the rationale of problem solving. This understanding can help teachers provide more and higher quality problem solving experiences for children. Students can also benefit from using the model. They may be more likely to be enthusiastic about working problems if they have a guide and teacher help in dealing with the most difficult part of solving problems — knowing what to do.

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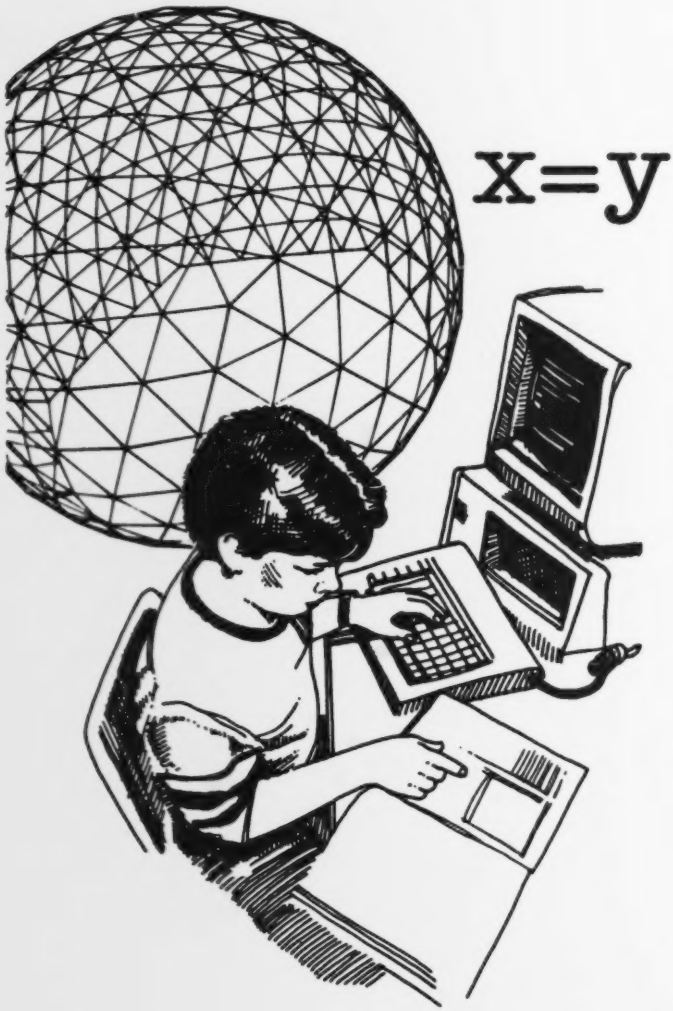

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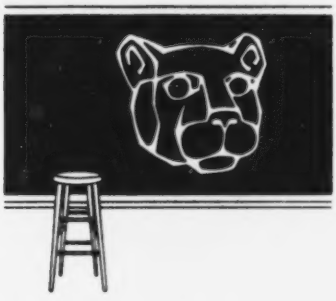

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LEARNING AND TEACHING MATHEMATICS THROUGH COMPUTING: LESSONS FROM A PROGRAM FOR MATHEMATICALLY TALENTED MIDDLE SCHOOL STUDENTS*

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Mathematics educators are in singular agreement on two important issues concerned with the teaching and learning of mathematics. There is, today, little debate, first, on the place of problem solving at the heart of the school mathematics curriculum and, second, on the desirability of using computers as an integral school mathematics tool. During the past few years conferences of mathematics educators, computer educators, and cognitive psychologists have convened and produced thoughtful analyses on the issues of the integration of computers in mathematics education (NCTM, 1984), the current state of research knowledge about mathematical problem solving (Silver, 1985), and the state of the art in teaching general and content-specific thinking skills (Segal, Chipman, Glaser, 1985).

Concerns with teaching problem solving, the growing need to find ways to integrate computing in mathematics instruction and national attention currently centered on the teaching of mathematics at the middle school level found embodiment in a special mathematics education program for mathematically talented middle school students begun during the summer of 1985 on the University Park Campus of The Pennsylvania State University. The implementation of the program provided interesting insights into promising curriculum and methods for enhancing problem solving through the tool use of the computer.

The Program

The focus of the 1985 summer program was the implementation of a one-week residential program in computing and mathematics for mathematically talented middle school students. Unlike other summer programs for the gifted, the Penn State program incorporated theory-based curriculum development and preservice and inservice practicum experiences for teachers.

The program had two major facets:

1. Participants in a graduate level mathematics education course stud-

*Parts of the curriculum work here were supported in part by a grant from The Pennsylvania State University College of Education Alumni Society.

ied current theory about characteristics of the mathematically gifted (Krutetskii, 1976), and about curriculum for the gifted. They explored sophisticated computer programs (function graphers, symbol manipulators) and designed mathematics curricula which used the computer as a tool.

2. Under the tutelage of experienced teachers of the mathematically gifted, middle school students who were enrolled in a one-week residential program explored mathematics and computing in intensive 27-hour courses.

Several opportunities were offered within the context of these two inter-related programs for the exchange of ideas and expertise among participants. Participants in the mathematics education course met with teachers of the middle school course to share expertise and critique curriculum ideas. One of the participants in the mathematics education course exercised the option to engage in a practicum experience with an elective portion of the middle school program.

A program with as many interwoven pieces as this one provides the opportunity for analysis from a myriad of viewpoints. Perhaps the most informative lens through which to view the program is that of curriculum. Two questions of importance arise:

1. What are some of the guidelines that current knowledge about the problem solving abilities of the mathematically talented suggests concerning the nature of appropriate curricula?
2. What are some of the ways that experienced teachers of the mathematically gifted implement those curricula?

Curricula for the Mathematically Talented

A remarkably indepth study by the Soviet psychologist, V.A. Krutetskii (1976), provides the mathematics educator with a coherent picture of the range of abilities possible in mathematically talented youth. An analysis of the implications of Krutetskii's findings appears elsewhere (Heid, 1983). It is informative, however, to focus on some of the more pervasive elements of Krutetskii's theory.

Very talented mathematics students, according to Krutetskii, have a tendency to look at problems through a different lens. They are quick to see the structure of a problem, they deal readily with the abstract, and they generalize problems and solutions almost immediately. They reason rapidly and are able to change the direction of their reasoning without confusion. They frequently come to correct conclusions without having seemed to reason through the "logical" steps and are intrigued by the search for an elegant solution. The mathematically inclined students with whom most teachers are familiar do not seem to possess these abilities to as great an extent as the subjects in Krutetskii's study. The characteristics, nonetheless, are familiar ones, and provided a basis for sound curriculum development in the summer program.

Topics chosen for study by the mathematically talented must, above all, be robust. The talented student can abstract and generalize and see connections between concepts. Topic areas should provide them the opportunity to exercise these talents. Topics developed in the aforementioned mathematics education course included taxicab course geometry, patterns in Pascal's triangle, and connections among the binomial theorem, Pascal's triangle, and the Fibonacci sequence (See Note 1).

One general area of mathematics introduced in the middle school program and which has particular promise for the mathematically talented middle school students is number theory. Concrete applications of GCD and LCM arise (See Note 2) in a surprising variety of understandable and intriguing problems. Explorations of number patterns are made even more exciting and accessible with the use of sophisticated symbol manipulation programs like muMath (Heid, 1985). When mathematically talented students are introduced to problem areas which rely on basic concepts they understand, they often seek and find more patterns than the curriculum writer has identified. Finding topics with potential for rich exploration is an important curriculum task and one which is often forgotten in the usual curriculum rush toward Calculus.

In the exhilaration of searching for curricula which will enable the mathematically inclined to use their talents, one must not fail to recognize that these students need work in developing certain problem solving skills. The middle school course on problem solving challenged students with a broad variety of accessible, and interesting problems (See Note 3). As students tackled one problem after another, they used and developed skills in organizing data, in looking for patterns, in recognizing the capacity of mathematical analysis to expand one's thought beyond the physical, in estimation, and in analysis of the seemingly impossible. As reported by one student, they developed and refined strategies for approaching problems they had never seen before.

Techniques for Delivering the Curriculum

Just as important as the choice of curriculum are the techniques used to convey the course ideas. What are some of the special techniques in computer enriched mathematics used by successful teachers of the mathematically gifted? Classroom observations of the middle school classes (See Note 4), filtered by seven years of experience in similar programs, provided the writer with summary perceptions of promising pedagogical techniques for work with the mathematically talented middle school student. Underlying these suggestions is the recognition that the teacher him/herself must be flexible (willing to take risks), broadly knowledgeable about the subject matter and its connections to other parts of mathematics, and genuinely interested in problem solving. The teacher must, above all, be visibly enthusiastic about the mathematical ideas he or she is introducing to the students.

Some observation-based suggestions for computer-based pedagogical techniques for working with the mathematically talented middle school student include:

1. When starting a new problem, make sure students have a firm understanding of the basic definitions involved. Draw the definitions from the students. Challenge them to supply and defend definitions of concepts they have encountered ("even," "odd," "factor," "repeating sequence"). Have other students argue for or against proposed definitions. Offer disconfirming or extending examples. Remind them of possible inconsistencies. Approach the task as a joint teacher-class exploration of the concept. This technique will draw on the strong intuition these students seem to have and help them cross the bridge to formalization.
2. Use the computer as a tool for the generation of data only after students have a solid understanding of the problem at hand. When students are using packaged or teacher-made programs, challenge them to find the data which provides the longest or shortest computer output. Since levels of mathematical talent vary widely even within classes like these, it is important to design activities through which each student can experience a personal success. Challenges should be open-ended enough so that a variety of answers are appropriate.
3. Encourage students to design experiments for gathering computer data. If multiple input is required, advise them on techniques of holding some values constant. Ask what happens when the input is large or small or prime or even or ... . The computer is an ideal tool for helping students refine their problem-solving strategies.
4. Introduce problems in the context of physical settings. Although many of these students are good at abstraction, others are only beginning to develop formal reasoning. Concrete representations provide both of these groups needed grounding for the abstractions.
5. Use the computer in a whole class setting to verify student prediction of outcomes. Mathematically talented students are often reluctant to guess for fear of getting teacher disapproval for wrong answers. They should be encouraged to guess, and the computer can be used for quick and nonpersonal verification of answers.
6. Often a student will come up with an answer more quickly than the rest. Try asking him or her not to give the answer but to tell what he/she looked at to help in starting to see the answer. This will give others some insight into alternative ways of looking at the problem without bringing premature closure. It will serve the student giving the explanation by encouraging metacognitive activity.
7. When students start gathering computer data in looking for a

pattern, ask them to look for particular classes of input or of answers (e.g., even, relatively prime).

8. Start some problems by giving students an example of a relationship and asking them to generate another example. Verify examples with a computer run. Mathematically talented students often generalize immediately. Asking for a second example gives the generalizer the opportunity to verify understanding, while giving others the chance to formulate and refine their own generalizations.
9. When an initial problem is solved, ask students to consider what happens if problem conditions are relaxed or restricted. Such changes in conditions can be built into the computer program to provide ready corroboration of new "theorems."
10. Challenge students to discover their own "theorems" about basic problems. Such challenges capitalize on and build the sense of mathematical structure that seems to be natural in good problem solvers.
11. Make occasional brief reference to related mathematics beyond what is being explored. You need not explain in detail all the mathematics to which you refer. Let students know that connections exist.
12. Share with students the ways you have learned about mathematics — books, talks, and most importantly, "messing around." Students need to recognize mathematics as a process.
13. Point out the mathematical nature of what students themselves are doing to solve problems: gathering data, conducting experiments, testing intuitions, etc.

These topics and techniques enabled teachers in the 1985 summer program to turn middle school students on to mathematics beyond the regular school curriculum. The methods and mathematical ideas played out by this special group of teachers and students provide some ideas that talented teachers can adapt to more traditional school settings. (See Note 5)

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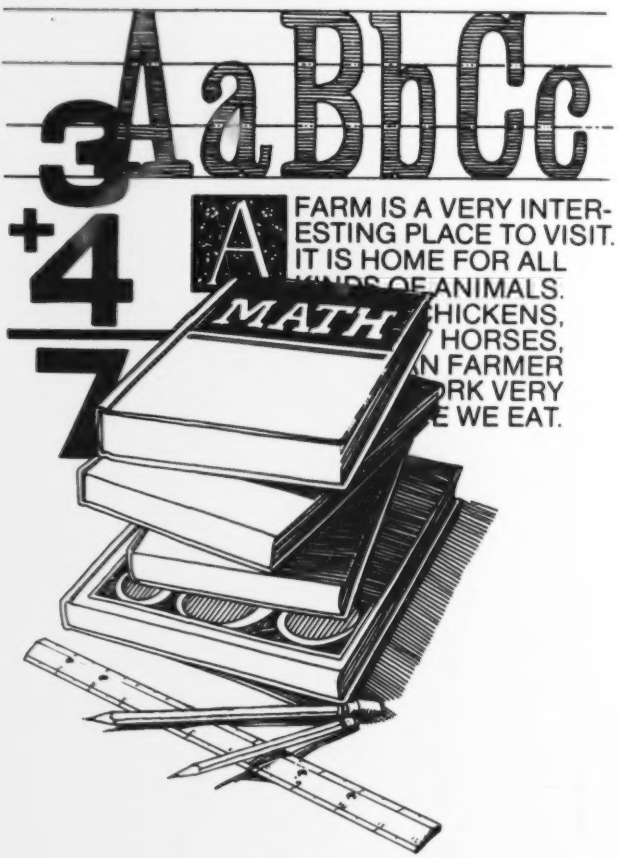
NOTES

1. Topics cited were developed by L. Kilaceski, W. Azzara, and L. Ostroski.
2. Curriculum cited was developed and implemented by S. Clarke.
3. Curriculum cited was developed and implemented by J. Larsen.
4. Classes were taught by S. Clarke and J. Larsen.
5. For details about The Pennsylvania State University Summer Programs for Mathematically Talented Middle School students, contact the author at 178 Chambers Building, University Park, PA 16802



MATH WITH A TWIST

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With the recent emphasis on mathematics and science, the Governor of Pennsylvania recommended that graduation requirements in these areas be increased statewide. Many teachers anticipated a great deal of curriculum writing during the summer of 1985. This was not the case at Penn Wood High School in Lansdowne, Pennsylvania. We scrutinized our curriculum three years ago and decided changes were essential.

Although the course offerings reflected levels typically seen in most high schools (namely general, academic, and accelerated), we were not adequately meeting the needs of all of our students. Placement into the courses was determined by teacher recommendation, testing, and counselor input, in conjunction with the student's future plans. However, many students were dropping math after completing the general course in tenth grade. To remedy the situation and to encourage more students to continue their studies, new courses were developed and implemented. Figure 1 illustrates the courses and their relationships.

Beginning in ninth grade "Basic Algebra 1" is studied. Not as rigorous as the traditional Algebra 1 course, topics are introduced at a more leisurely pace. The book *Basic Algebra* by Brown, Smith, and Dolciani is less threatening than the one utilized by the academic students. Basic skills are constantly reinforced as algebra is taught. The first year concludes with the introduction of the various types of factoring.

In tenth grade, "Basic Algebra/Geometry" continues the sequence. It is here that traditional Algebra 1 is completed and a final examination administered in January. During the second semester "Basic Geometry" commences. Not as demanding as the regular geometry, students are introduced to the concept of proof in a less formal way. Activities-oriented, students study deductive reasoning by use of comic strips, puzzles, deciphering codes, solving mysteries, etc.

By the end of eleventh grade, "Basic Geometry" is completed. In this sequence, a student fulfills the algebra and geometry requirements in three years rather than two. For those opting to continue their study of mathematics, "Basic Algebra 2" provides an avenue for exploration. Topics include those of the traditional Algebra 2 course, minus proof. The *Holt Algebra 2* text was selected due to its ease of comprehension and its variety of exercises.

Because of the popularity of these courses, "Basic Trigonometry" was added in September, 1983. A full year course offered to seniors, the first quarter is spent reviewing and reinforcing Algebra 2 topics. Three-

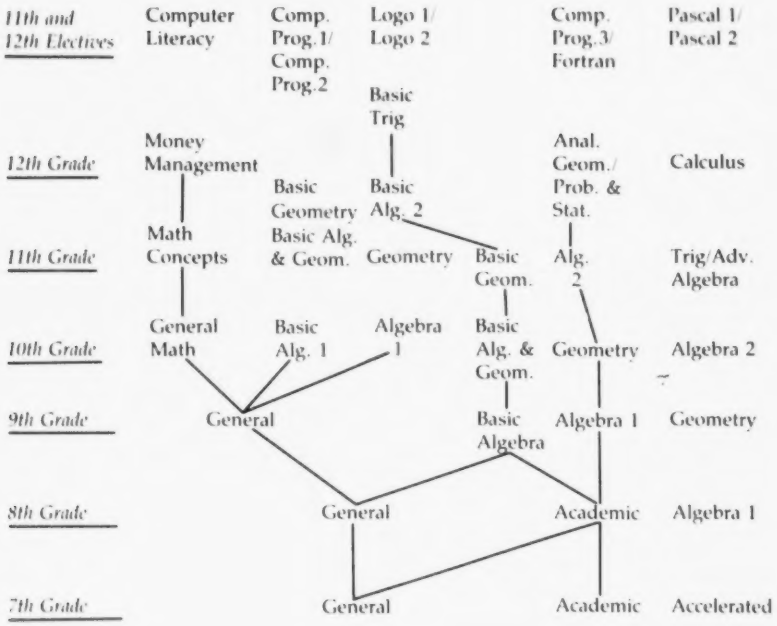

Figure 1
Secondary Mathematics Sequence
Penn Wood High School

fourths of the year concentrates on trigonometry.

The staff at Penn Wood has found these courses to be advantageous for several reasons. First, students who could not achieve success in academic classes are completing several courses in this sequence. Those who would have experienced boredom in General Math are being challenged. In addition, students are electing more math than required by the school district for graduation.

If a student performs well at this level, his teacher may recommend him for a course in the traditional sequence. Therefore, incentives are built into the program. Conversely, if a student experiences difficulty at the academic level, he may be placed in the "Basic" sequence temporarily. The advantage is that students remain in math classes rather than wasting time in study halls.

Although the "Basic" sequence was very successful, one segment of our population still dropped mathematics after a year of General or Business Math. For this reason we instituted several semester courses.

Open to all students but designed for those with below average math ability, "Money Management" is a popular elective. Topics include: saving and investing money, real estate financing, insurance, use of

credit cards, installment plan buying, and costs of purchasing and maintaining automobiles. Calculator activities are incorporated into every topic.

"Math Concepts," another elective, has become a favorite among our students. They are expected to "build" their dream house. Based on the premise that money not affect their decisions, students plan, design, and furnish the house they would like to own. They are provided with price lists of materials to guide them in their selections. When computing costs of their project, they utilize basic math skills. Use of the calculator is encouraged.

To expose general students to probability, "Gaming and Game Simulation" was devised. In addition to the rules of games of chance, students learn to compute the probability of occurrence of each event. The course objectives state the necessity for handling fractions, decimals, and percents with ease.

A one-semester elective, "Computer Literacy," appeals to most general students. This course affords them the opportunity to acquire hands-on experience with the microcomputer. Only a slight emphasis is placed on programming thus allowing all students to experience success. Upon completion of the Literacy course, many students wish to continue their work with microcomputers. Our electives were expanded to include "Logo 1" and "Logo 2" to satisfy this need.

The first semester emphasizes "turtle graphics." Students learn the Logo language as they control the computer. The turtle becomes "an object to think with." (Papert, 1980) "Logo 2" introduces list processing. Students complete projects which incorporate the graphics learned during the first semester with words, lists, and numbers.

Since the inception of the elective courses, enrollment in mathematics classes at Penn Wood High has increased. The creativity of the teachers is a key factor to their success. Textbooks are not provided since none met our specific needs. Therefore, teachers, not publishers, determine the curriculum. Only a minimal amount of homework is assigned. Most of the work is completed in class while teachers supervise.

The interest and enthusiasm demonstrated by the teachers is appreciated by the students. This, coupled with the variety of activities comprising the content of the courses, has resulted in few, if any, discipline problems. In our experience, considering the needs of the general students has produced positive results. We feel our students are well-prepared to accept their future mathematical challenges.

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Scott, Foresman High School Math

Pre-Algebra
Algebra: First Course/Second Course
Geometry, Second Edition
Trigonometry, Second Edition
Precalculus
Mathematics in Life, Second Edition
Consumer and Career Mathematics, Second Edition

a tradition to trust
Scott, Foresman
Oakland, NJ 07436

MICROCOMPUTERS, MONTE CARLO, AND MATHEMATICS

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Rolling dice, throwing darts, spinning pointers, dealing cards, flipping coins — fun for all, but certainly not related to any serious academic or practical pursuit. Or are they?

In scientific circles, the names John von Neumann and Stanislaw Ulam conjure up thoughts that are far from frivolous. While trying to solve an extremely complicated problem at the Los Almos Scientific Laboratory during World War II, those mathematicians developed a method of solution which they gave the secret code name "Monte Carlo." Their problem of determining how far neutrons would penetrate various materials was solved using the equivalent of a roulette wheel to determine probabilities (Keating, 1985).

In the Monte Carlo method, simulations generate random numbers to solve a problem. This versatile technique has been applied to many logistics problems and scheduling studies, such as planning baggage handling facilities at the Dallas-Fort Worth Airport and determining the average length of time a bank customer must wait in line to use an automatic teller machine (Keating, 1985).

Few subjects provide the secondary school mathematics classroom with a richer blend of history of mathematics, problem solving, computer programming, theoretical mathematics, practical applications, and amusement — than probability. (See Berlinghoff, 1968, for a brief history of probability.) And the Monte Carlo method can be an exciting place to start.

Generating random numbers and examining them for randomness is, in itself, an intriguing activity leading to equidistribution tests and checks for "runs." The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, ... would pass an equidistribution test, but they obviously fail any "runs" test for "patternlessness," and hence, are not random. (For further information on randomness and computer programs for generating random numbers, equidistribution tests, and runs tests, see Kimberling, 1984, or Duncan and Litwiller, 1981).

A related "barrel of monkeys" is how long it would take one monkey seated before a typewriter and allowed to poke randomly at the keys to type "OFF WITH HER HEAD" (which the Queen of Hearts screeched at Alice in Wonderland). For an easy explanation of why it would take 12 million monkeys 10 billion years to produce OFF WITH HER HEAD, see Carlson (1985)!

In BASIC, the use of the RND (random) function makes Monte Carlo simulations easy to run. The expression RND (1) generates a decimal between 0 and 1, RND(1)*2 generates a decimal between 0 and 2, and, using the greatest integer function, INT(RND(1)*2) generates randomly an integer X, where $0 <= X < 2$, that is $X = 0$ or $X = 1$. If we assign a "head" to the occurrence of a 0 and a "tail" to the occurrence of 1, then the single line INT(RND(1)*2) in a computer program will "toss a coin." Similarly X = INT(RND(1)*6 + 1) will "throw one die," that is, randomly generate 1, 2, 3, 4, 5, or 6; and two such lines for X and Y will "throw two dice."

The ideal setting in which to use the computer as a tool to teach probability is a classroom in which each two or three students is given the supplies to perform the real-life trials of each experiment (e.g., dice, coins, cards). This is followed by simulation of the experiment on a computer, and the mathematical solution of the problem directed by the teacher. If a high student-computer ratio is impossible, even one computer per class can elicit exclamations of surprise and satisfaction.

Students with no previous introduction to probability or computers can be guided through the use of a worksheet and simple programs on diskette. The following format, used in the problems which follow, is an effective structure for conducting the classroom activities: A. Statement of the Problem, B. Real-Life Trials, C. Computer Program and Simulation, and D. Mathematical Solution. Many interesting problems can be simulated using the Monte Carlo method; and the class' satisfaction is immense when the real-life trials, computer simulation results, and mathematical solution jibe.

The following problems are presented to entice the reader into sampling the application of the microcomputer to probability, beginning with the Monte Carlo technique. (If you become "hooked," the author will be glad to send you a copy of a diskette containing other probability and statistics programs which need not be used exclusively in a probability course.) Depending on the emphasis placed on the programming skills and mathematical solution, they can be included for enrichment or motivation in various mathematics classes such as algebra I & II, college algebra, precalculus, and calculus.

After appreciating the use of the computer in studying probability, instead of remarking that "Mathematics is the handmaiden of science;" one is justified in making the twentieth-century update that "The computer is the handmaiden of mathematics."

PROBLEM 1: Catching the Counterfeiter

A. *Statement of the Problem:* A king suspects that his treasurer is robbing him by substituting counterfeit coins for gold ones. The coins are packed 25 to a bag, and, in fact, the treasurer has been placing one false coin in every bag. The king commands that he be brought 25 bags of gold. He will

select one coin from each bag to be analyzed for metal content. What is the probability that the king will find a counterfeit coin? (Lappan and Winter, 1985)

B. *Real-Life Trials:* Each pair or group of students is given a brown lunch bag containing 25 plastic bingo chips, 24 green and 1 red (available at a 5 & 10 cent store). One "coin" (chip) is drawn from the bag and observed to be "real" or "counterfeit" (green or red). The coin is replaced in the bag and the drawing is repeated for a total of 25 times or until the red coin is drawn, whichever occurs first. The trial is considered successful and completed if the red coin is drawn during the 25 or less drawings. The class' total number of successful trials divided by its total number of trials gives the detection probability.

C. *Computer Program and Simulation:* The class "walks through" the following computer program. The extent of the discussion is determined by the programming experience of the students (and teacher). If the number 12 is generated by line 50, we have found a false coin. Since one false coin proves the guilt of the treasurer, the trial is terminated, and the treasurer is beheaded.

```
10 INPUT "NUMBER OF TRIALS "; N
20 D = 0: REM D MEANS TREASURER IS DETECTED
30 FOR I = 1 TO N
40 FOR J = 1 TO 25
50 P = INT(RND(1)*25 + 1)
60 IF P = 12 THEN 80: REM BAD COIN DRAWN
70 NEXT J: GOTO 90
80 D = D + 1
90 NEXT I
100 PRINT "DETECTION PROBABILITY: "; D/N
```

Five runs of the program follow:
NUMBER OF TRIALS? 100
DETECTION PROBABILITY: .61, .66, .66, .61, .57

D. *Mathematical Solution:* The probability of drawing a false coin from the first bag is 1/25. Hence the treasurer's probability of escaping detection when the first bag is opened is 24/25. The probability of escaping detection for two bags is (24/25)*(24/25), and the probability of being safe through 25 bags is

$\left(\frac{24}{25}\right)^{25} \doteq .36$. Hence probability of being caught is $1 - .36 = .64$.

[An interesting extension of this problem, appropriate for a calculus class, occurs when more coins are placed in each bag and, correspondingly, more bags are examined. If N bags are examined, each containing N coins, then the probability that the treasurer escapes detection is

$\left(\frac{N-1}{N}\right)^N = (1 - 1/N)^N$ which approaches 1/e as N approaches positive infinity. Hence for large N, the probability of detection is $1 - 1/e \doteq .63$. (Lappan and Winter, 1985)

PROBLEM 2: The Duelling Marksmen

A. *Statement of the Problem:* Two marksmen have a duel. They each have a 50 per cent chance of hitting the target on each shot. What is the probability that the first man to shoot wins the duel? (Hinders, 1981)

B. *Real-Life Trials:* Each student needs one coin and a tally sheet with two columns labeled "First Person Wins" and "Second Person Wins". The student begins flipping the coin. A head is considered a "hit" and a tail a "miss".

If a head appears on the first flip, the first person wins the duel and a tally mark is made in the appropriate column. If the first toss is a tail, the second person gets a shot, i. e., the second flip. If the coin comes up heads, the second person wins and the win is recorded in the appropriate column. If a tail comes up, the coin is tossed a third time, etc.

After the class has simulated several hundred duels, the data are combined and the number of wins by the first person is divided by the total number of duels.

C. *Computer Program and Simulation:* In the following program, 20 duels are performed each time the program is executed (see line 230). The percentage of duels won by the first marksman is given by line 260.

```
10 N = N + 1: HOME : VTAB (8)
20 PRINT "DUEL   FIRST MARKSMAN   SECOND MARKSMAN"
30 LET X = RND (1): IF X > .5 GOTO 70
40 PRINT N, "HIT"
50 F = F + 1: GOTO 150
60 GOTO 150
70 PRINT N, "MISS"
80 LET X = RND (1)
90 IF X < .5 GOTO 130
100 PRINT " "," ","HIT"
110 S = S + 1
120 GOTO 150
130 PRINT " "," ","MISS"
140 GOTO 30
150 VTAB (20): PRINT "PRESS 'RETURN' ..."
160 GET G$
170 HOME : VTAB (8)
180 PRINT "THE FIRST MARKSMAN HAS WON ";F;" DUELS"
190 PRINT : PRINT "AND THE SECOND MARKSMAN HAS WON ";S
200 PRINT : PRINT "DUELS.": PRINT : PRINT : PRINT
```

```
210 PRINT "PRESS 'RETURN' FOR THE NEXT MATCH."
220 GET G$
230 IF N = 20 GOTO 250
240 GOTO 10
250 HOME : VTAB (8)
260 PRINT "THE FIRST MARKSMAN HAS WON ";F * 5; "%"
270 PRINT : PRINT "OF THE TIME."
280 END
```

(Collis, 1982)

The following results from the 9th duel in one run of the program shows that the second marksman won on his second shot:

DUEL	FIRST MARKSMAN	SECOND MARKSMAN
9	MISS	MISS
9	MISS	HIT

One complete run of the program (20 duels) gave the following printout: THE FIRST MARKSMAN HAS WON 12 DUELS AND THE SECOND MARKSMAN HAS WON 8 DUELS. THE FIRST MARKSMAN HAS WON 60% OF THE TIME.

D. *Mathematical Solution:* The first marksman has the probability of ½ of winning on the first shot. If he misses, his probability of winning on his next shot must be determined. This assumes that the second marksman misses his first shot. Hence the probability of the first marksman winning on his second shot is ½ * ½ * ½. Similarly, the first marksman will have the opportunity to win on his third shot only if all the preceding shots were misses. Hence the probability of the first marksman winning on his third shot is ½ * ½ * ½ * ½ * ½. This reasoning continues, so the probability of the first marksman winning the duel (that is, winning on his first shot or his second shot or his third shot, etc.) is

$$\tfrac{1}{2} + (\tfrac{1}{2} * \tfrac{1}{2} * \tfrac{1}{2}) + (\tfrac{1}{2} * \tfrac{1}{2} * \tfrac{1}{2} * \tfrac{1}{2} * \tfrac{1}{2}) + \ldots$$
$$= \tfrac{1}{2}[\, 1 + (\tfrac{1}{4}) + (\tfrac{1}{4})^2 + (\tfrac{1}{4})^3 + (\tfrac{1}{4})^4 + \ldots \,]$$

which is a geometric series with first term of ½ and common ratio of ¼. The sum of a geometric series

$$\sum_{i=1}^{\infty} a * r^{i-1}$$

is a /(1-r). Applying the formula to our problem, where a = ½ and r = ¼, the probability that the first marksman will win is

$$\frac{\tfrac{1}{2}}{1 - (\tfrac{1}{4})} = \tfrac{2}{3} \doteq .667$$

(Hinders, 1981).

Additional problems, including their computer simulation programs and mathematical solutions, can be found in the following references.

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HOW TO KICK THE T-TABLE HABIT

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Graphing functions is one of the fundamental activities in mathematics teaching and learning, and is a topic that can be found in all levels of high school mathematics. Students and teachers have been plotting points to analyze functions ever since Rene Descartes discovered analytic geometry by watching a fly crawling around the ceiling of his room. Plotting points and creating graphs tends to be one of the favorite activities of mathematics students. Everyone can do it! Pick an x, figure out y, plot a point, pick an x, figure out y, etc. Ask a high school student to graph $y = x^2$ and there is a better than even chance that you will see a t-table ... pick an x, figure out y. Students love it! "Every function has a graph," is replaced by, "every function has a t-table." T-tables are so thoroughly imbedded in our young people's minds that even the study of calculus with first and second derivative tests for minimums, maximums and concavity does not alter the inevitable t-table. Ask a calculus student to graph $y = x^3 + 3/x$ and chances are you will see a t-table. Pick an x, figure out y....

It is difficult to find fault with a process that seems to be such a favorite part of teaching and learning mathematics. Students are happy to be making t-tables because they are "learning by doing." Teachers are glad to have an activity where every student can participate and be involved. Whatever the defense of using t-tables might be, such a rationalization tends to perpetuate a practice for all the wrong reasons. Of course, t-tables are not all bad. What is regrettable is that it seems that when high school students think about graphing a function it is more like graphing a t-table. The emphasis of the student and teacher should be on the graph. When the student tries to visualize $y = x^2$, he or she should see a graph, not a t-table.

How can we kick the t-table habit? The answer is simple. Students and teachers have to put more emphasis on the final product. T-tables have a place in constructing graphs but students need more practice in the desired result — relating functions to their graphs. Constructing a t-table by hand takes too long. The computer can help. Let the computer figure out the values for the t-table. Even better ... have the computer plot the points. Then the student can concentrate on what happens to a function when changes are made like $y = x^2$, $y = x^2 + 2$, and $y = x^2 - 2$. Graphing with the help of a computer emphasizes creativity and the inherent beauty of the finished product. Students and teachers will continue to like graphing but for all the right reasons.

There are many examples of good graphing programs available for all of the popular microcomputers. Sometimes these programs do more than the teacher needs or even cares about. At Upper St. Clair we use a relatively simple program created and modified for three microcomputers, Apple, Atari and IBM. The program which we call GRAPH was written by students Blaze Stancampiano and Eric Graeler. Some of the ways that we have found this program to be useful are:

When there is one computer available for the classroom:

1. Give students approximately three minutes to sketch the graph of a function. Then show the computer result.
2. Display a computer graph, ask the students what function is represented.
3. Work on variations of one function. Ask students what happens to $y = x^2$ when $y = x^2 + 5$; then show the computer graph.
4. Display a graph. Ask where are the minimums, maximums, inflection points? Then try to determine what happens at these points.
5. Ask students to be creative. Think up a variation on a common function that maybe nobody has ever seen before like $y = \sin(x^2) + 5\cos(x)$.
6. The computer should be a resource for a quick sketch of a function that the student may want to check in the context of a larger problem.

When a group of computers is available in a computer resource center:

7. Give the students lots of practice in entering functions and making sketches directly from the monitor screen.
8. Encourage students to use the graph program any time they want to check the sketch of a function.
9. Develop work sheets which will help students to achieve independently the objectives considered above in a one computer/ classroom situation.

Strategies such as these will help to overcome the problem of too much emphasis being placed on the process of making t-tables to find the points. Students need to comprehend the process, not just become good at making t-tables. The emphasis ought to be on the end result, which is a sketch of the function. A graphing program can provide students and teachers with increased insight into the beauty and excitement of mathematics.

The following figures give examples of program GRAPH for each of the three systems — Apple, Atari, IBM. The function is always entered on line 180 as y in terms of x. If you desire to change the scale for the IBM or Apple version, change line 184 according to:

IBM		Apple	
s = 1	(1 block = 20 units)	s = 2	(1 block = 5 units)
s = 5	(1 block = 4 units)	s = 5	(1 block = 2 units)
s = 10	(1 block = 2 units)	s = 8	(1 block = 1 unit)

The Atari version will set the scale as part of the initial screen request. Also press START on the Atari when you want to keep the same function but change the scale.

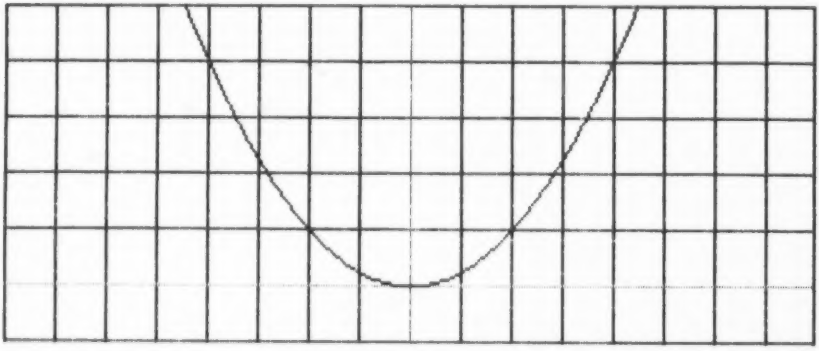

FIGURE 1

]List

```
10 REM APPLE IIe FUNCTION PLOTTING PROGRAM
20 REM GRAPHICS MODE
30 HOME : HGR : HCOLOR= 2
40 REM PUT MARKINGS ON THE AXIS
50 FOR X = 0 TO 279 STEP 10
60 HPLOT X,0 TO X,159
70 NEXT X
80 FOR Y = 0 TO 159 STEP 10
90 HPLOT, 0,Y TO 279,Y
100 NEXT Y
110 HPLOT 0,159 TO 278,159 TO 278,0
120 HCOLOR= 3
130 HPLOT 140,0 TO 140,159
139 HPLOT 141,0 TO 141,159
140 HPLOT 0,80 TO 279,80
145 E = 1
150 REM PLOT THE FUNCTION
160 FOR X = -14 TO 14 STEP .1
165 IF X = 0 THEN 200
170 X1 = 140 + X * 10
180 Y = X ^ 2
184 S = 10
185 Y = 80 -Y * S
186 IF Y > 190 OR Y < O THEN E = 1: GOTO 200
```

```
187 IF E = 1 THEN HPLOT X1,Y:E = 0
190 HPLOT TO X1,Y
200 NEXT X
210 GOTO 210
```

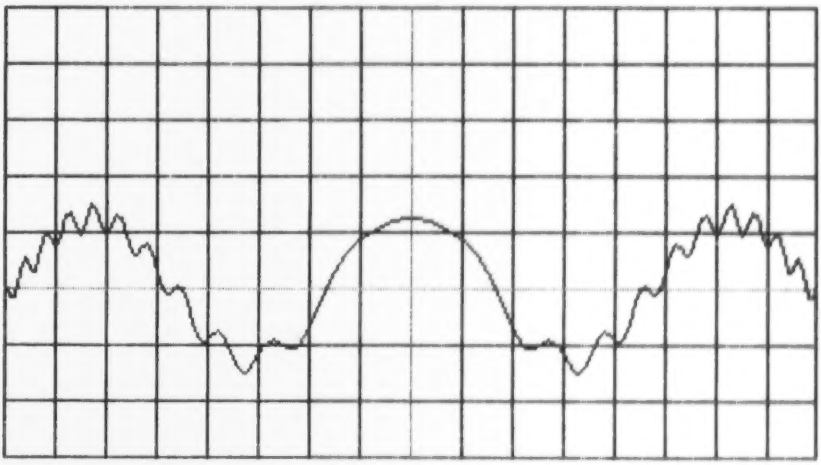

FIGURE 2

```
10 REM IBM FUNCTION PLOTTING PROGRAM
20 REM GRAPHICS MODE
30 SCREEN 1:COLOR 0,5:WIDTH 40:CLS
35 KEY OFF
40 REM PUT MARKINGS ON THE AXIS
50 FOR X=0 TO 319 STEP 20
60 LINE (X,0)-(X,200),2
70 NEXT X
80 FOR Y=0 TO 199 STEP 20
90 LINE (0,Y)-(320,Y),2
100 NEXT Y
110 LINE (0,100)-(320,100),1
120 LINE (160,0)-(160,200),1
130 LINE (0,199)-(319,199),2
140 LINE (319,0)-(319,199),2
145 E=1
150 REM PLOT FUNCTION
160 FOR X=-8 TO 8 STEP .1
165 IF X=0 THEN 200
170 X1=160+X*20
180 Y=SIN(X^2)+5*COS(X)
```

```
184 S=5
185 Y=100-Y*S
186 IF ABS(Y)>32767 THEN E=1:GOTO 200
187 IF E=1 THEN PSET(X1,Y),3:E=0
190 LINE -(X1,Y),3
200 NEXT X
210 GOTO 210
```

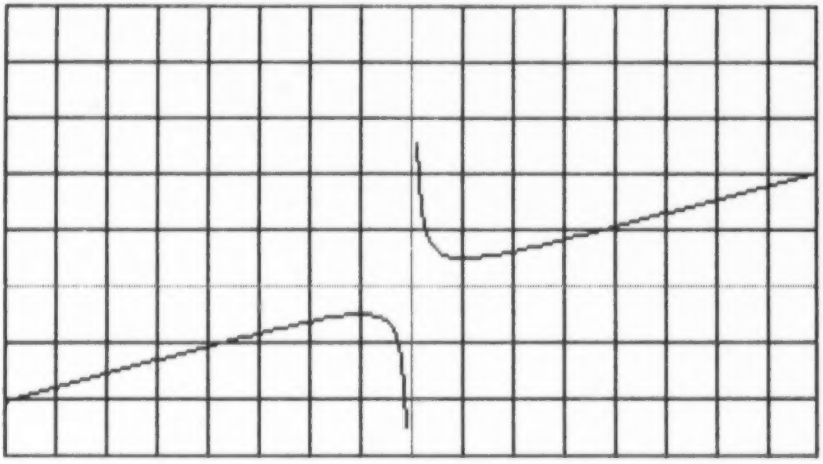

FIGURE 3

```
0 DIM Y(4): REM ATARI FUNCTION PLOTTING PROGRAM
1 TRAP 1:GRAPHICS 0:? :? "What is the (x) scale"
2 POSITION 10,5:? " ":POSITION 10,6:? "-3-2-1 0 1 2 3"
3 POSITION 10,9:? " ":POSITION 10,10:? "-9-6-3 0 3 6 9":POSITION
  24,1:INPUT XS
5 TRAP 5:GRAPHICS 0:? :? "What is the (y) scale"
6 TRAP 5:POSITION 10,5:? "      ":FOR I=1 TO 5:? "      I+D,85:NEXT
  D:NEXT I
95 FOR I=A+(10*40)+20 to A+(40*177) STEP (14*40):POKE I,73:NEXT I
96 I=A+(94*40):FOR D=1 TO 39:POKE I+D,170:NEXT D:COLOR
   1:PLOT 164,10:DRAWTO 164, 177:COLOR 0:PLOT 319,0:DRAWTO
   319,191
97 COLOR 1
160 FOR X=-11 TO 11 STEP 0.1
163 IF PEEK(53279)=6 THEN 1
165 IF X=0 THEN 200
170 XST = 160 + ((X-0.1)*14/XS)
171 XEN=163+(X*14/XS)
```

```
175 FOR CAL=1 TO -1 STEP -2:X=X+(CAL/10)
176 TRAP 200
180 Y=X+1/X
185 Y(CAL+3)=94-(Y*14/YS)
186 NEXT CAL
190 TRAP 192:PLOT XST,Y(2):DRAWTO XEN,Y(4)
192 TRAP 200:PLOT XST+1,Y(2):DRAWTO XEN+1,Y(4)
195 IF PEEK(53279)=6 THEN GOTO 1
200 NEXT X
300 IF PEEK(53279)=7 THEN 300
310 GOTO 1
375 FOR CAL=1 TO -1 STEP -2:X=X-(CAL/10)
376 ? X
386 NEXT CAL
```

WHAT WILL BE LEFT AFTER THE FACTS HAVE BEEN FORGOTTEN?

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Occasionally as classroom teachers we have the tendency to blame the principal, the school board, the community, the "system," or society for the problems we face on a daily basis in education. Of course there are problems which are caused by a variety of factors, many of which we cannot change. However, let's concentrate on an area where we do have control and where we can make a difference — our teaching strategies. Are there other possibilities besides the "check homework, lecture on new material, and give an assignment" routine? Probably most of us agree with Madeline Hunter (McGreal, 1983, pp. 84-89) that we need to include independent practice in our lesson design, so the issue of homework and how it is checked will not be addressed.

Instead, let's focus on the presentation of new information. What force requires our main technique to be "I'll explain how to do the problems, do a few examples, and then let you try"? Is it the school's policy, or is it the result of our teaching the way we were taught? There is no denying the expository method is efficient. It does pass information to students quickly (well, at least to their notebooks) and there is a great amount of material to be covered. Indeed, there are times when the lecture/recitation method may well be the right one to use. The questions, however, are: Do we always want to use this method? Are there better ways to develop students' understanding and problem-solving abilities? Can we use other strategies and still cover the content needed for future topics and mathematics courses?

Consider the topic, squaring a binomial, which is covered in algebra courses. A brief explanation would probably include an application of the distributive property and the use of the FOIL method as a shortcut. This approach is fine and works for most students, but how many times have we had to ask, "Where is the middle term?" For some, the distributive property and the FOIL method are not enough to gain a basic understanding of the concept. It is possible some students need to have a picture in order for the idea to make sense. An alternative method to presenting

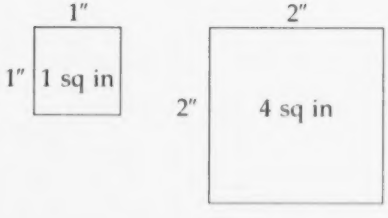
$$(a + b)^2 = a^2 + 2ab + b^2$$

as a fact to be memorized might be a guided discovery lesson.

For example the lesson could begin by giving the students rulers and then using the following steps:

1. Ask students to draw a 1″ and a 2″ segment.
2. Then have them draw two squares, one with 1″ on a side and the other 2″ on a side.
3. Ask students to construct a (1″ + 2″) by (1″ + 2″) square and subdivide it. (See Figure 1)
4. Compare the results in steps 2 and 3.
   (Note: There is a 1″ square and a 4″ square in both, but the drawing in step 3 has two additional rectangles which are equal in area.)
5. Ask students to draw conclusions.
   $(1 + 2)^2 = 1^2 + 2(1 \cdot 2) + 2^2$ or $(1 + 2)^2 \neq 1^2 + 2^2$

1.

2. 

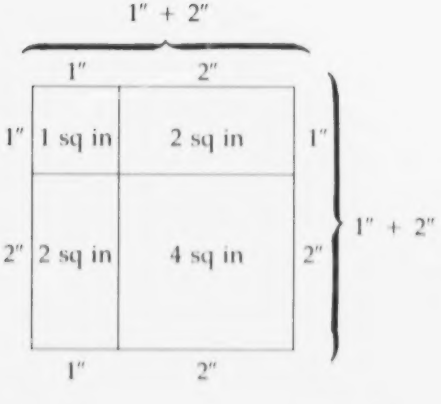
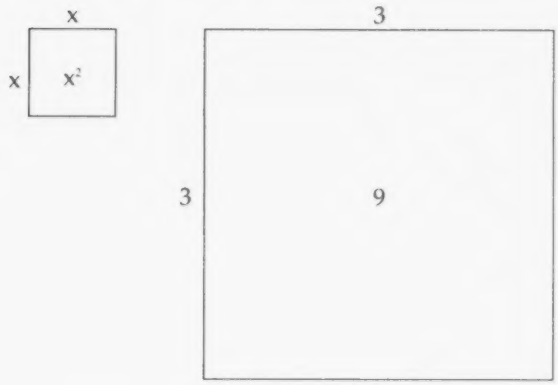
3. 

Figure 1

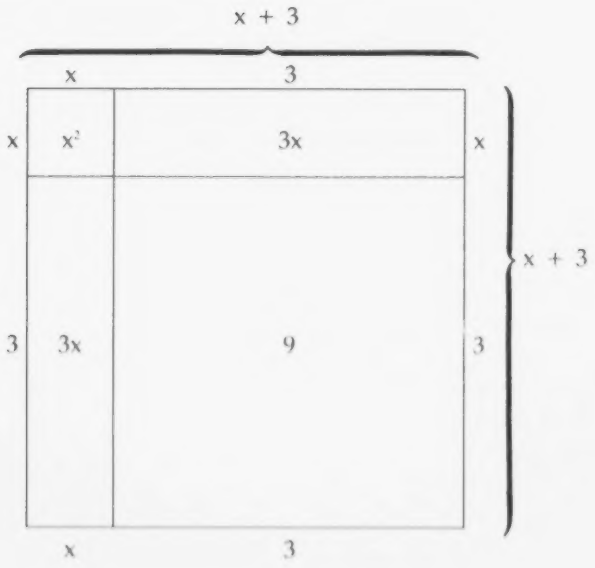
Next, extend the five steps to $(x + 3)^2$.

1. x 3

2.



3.



4. Note that the drawing in 3 contains two rectangles, each with an area of 3x.

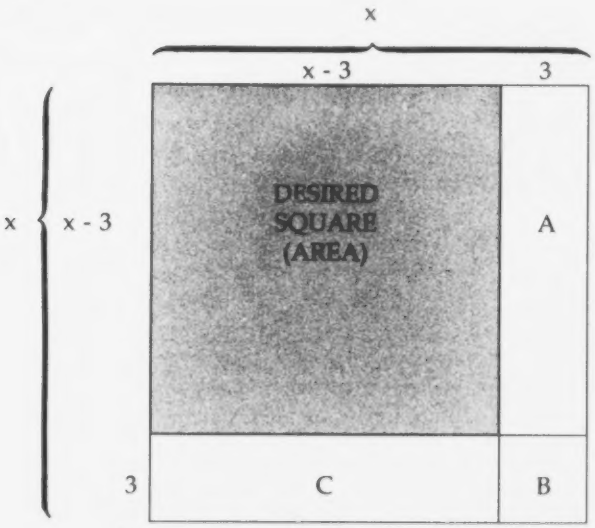
5. The conclusions can then be drawn.
   $(x + 3)^2 = x^2 + 2(3x) + 3^2$ or $(x + 3)^2 \neq x^2 + 9$.

This technique, with some variation, also can be extended to $(x - 3)^2$.

1. Draw segments of length x and 3.

x _____ 3

2.



The area of the entire square is x^2.
We want to subtract the area of A, B, and C.
The area of A and B is 3x.
The area of C and B is 3x.
Notice, we have included the area of B twice. Therefore, we need to add the area of B once to compensate for its being subtracted twice.
The area of B is 9.
Therefore,
$(x - 3)^2 = x^2 - 2(3x) + 9$.

Will this method take too much class time? That of course, is for each teacher to decide. However, what is twenty minutes (and perhaps a little noise) during class if it means that students can visualize and really understand that $(a + b)^2$ does equal $a^2 + 2ab + b^2$? You have helped your students move from a visual, relatively concrete representation to an abstract representation. See *Teaching and Learning Mathematics* by Fred Bell (1978, pp. 238-240) for a more complete description of beginning with

$(a + b)^2$, as well as an extension to $(a + b)^1$.

A geometry lesson on finding the sum of the interior angles of a polygon could be started by giving the students the formula, (n - 2)180, followed by numerous examples. Again, this is efficient, but is it meaningful? For some students, it probably is fine. For others, it is just one more fact to memorize until the test is over. If, instead, students were actively involved in "discovering" the formula, it would have more meaning for them and they would have engaged in higher level thinking by going through the process of trying to figure something out.

Depending on the level of the students and their prior knowledge, the lesson could begin by having the learners find that the angles of a triangle fit together to form a straight line having the measure of 180 degrees. This can be demonstrated by giving the students a triangle and having them fold the corners over and down showing that they meet to form a straight angle. If the students are older, the lesson could begin by having a student recall that the sum of the angles of a triangle is 180 degrees. Then, draw quadrilaterals, pentagons, and hexagons. Choose one vertex in each and draw from it as many diagonals as possible. Number the triangles which have been formed and find the sum of the interior angles of the polygon by finding the product of 180 and the number of triangles.

Have the learners record their data in a table and ask them to look for a pattern, form a hypothesis for the sum of the interior angles of any polygon, and then test their conjecture on an octagon. Instead of just adding one more fact to their knowledge base, the students have been involved in higher level thinking. They have been engaged in analyzing the situation, applying a previously learned fact to it, gathering data to use in forming a hypothesis, and then testing their hypothesis. Surely this whole process is more meaningful and worthwhile for the students than just being given a fact followed by time for practice on several examples.

Let's not forget to provide some variety in our teaching strategies for general mathematics students, too. Jerome Bruner's model called concept attainment (Joyce & Weil, 1980, pp. 25-46) could be used for finding the average of a set of numbers. Give students sets of numbers and their corresponding averages as examples of the concept along with several nonexamples. Depending on the grade and ability level of the students and whether this is a new idea or a review, the examples could be easy or difficult. Ask the learners to analyze each one and make conjectures regarding the relationship. Instead of having the students try to explain what they think the concept is, have them supply examples and nonexamples. For a class with a history of poor participation or for students who are unsure of their mathematical ability, emphasize that every guess is acceptable and that you will put the numbers under the proper heading if they so desire. In this manner, students can analyze the examples and nonexamples, make conjectures, and test them without the fear of being

embarrassed by giving wrong answers. Nonexamples are wanted just as much as examples since they are useful for testing hypotheses. When most of the students seem to have the concept, or if it seems as though many are frustrated by not having attained it, ask one of the students to explain what it is. Then take some time to have the learners discuss the thought processes they used in determining the concept.

There are many objectives in mathematics at all levels which can be attained through teaching strategies other than the lecture/recitation method. The content covered does not have to differ from what is normally included in any given course. What is different is the amount of active student involvement and the levels at which the learners must think. Instead of being presented with many facts to be memorized, practiced, and remembered for a short time, students are given tasks which require them to analyze the situation, transfer what they already know to this new situation, form and test hypotheses, and draw conclusions. They have been actively involved in higher level thinking. They have had to analyze instead of memorize. They have been encouraged to make conjectures. They have been involved in drawing the conclusions instead of being told. They have had to use and develop their problem-solving abilities. Perhaps there will be something left once the facts have been forgotten.

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DEVELOP YOUR OWN MATHEMATICS ACHIEVEMENT TEST

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Who knows your students or your curriculum better than you do? Certainly not a national testing group or the author of a textbook. Why, then, should the measurement of achievement in your classes be determined by a standardized test or a final exam that accompanies a given textbook? A test that measures a year's achievement in your class ought to exhibit a high degree of congruence with the written and taught curricula. By drawing on your knowledge as a teacher and by developing some test-item writing skills, you can develop a test that should be a better indicator of achievement in your curriculum than a standardized achievement test. You begin by building a basic test and then revise it annually to improve it and to adjust it to your curriculum changes.

Overview

Begin by using the written record (curriculum, including a topical outline, behavioral objectives, scope and sequence) of what is to be taught in the course. Use this record as a basis for writing test questions, which should be selected to test for understanding of all major concepts presented in the year's work. In general, use the multiple-choice format. It is effective in covering a substantial amount of material with objective results. (You might also want to include a few problems for which the student is required to show all of his/her work. When you score these few items, you'll gain extra insight into each student's ingenuity, organization of thought, and reactions to a situation which requires a sustained effort. Be sure to specify guidelines for scoring these problems so that you remain objective.)

Your test should probably have 50 or more questions in order to measure one year's learning. Writing so many test items for the first time will likely be time-consuming. But once they're finished, they can be used — some of them, at least — for years. New and better test items will occur to you as you work with your classes during the year — and if you record and file these, revising your test each spring will be easy. (Using a microcomputer to build a data base for expansion, analysis and revision could streamline the entire process of test development.)

Planning the Test

Before you develop individual test items, review Bloom's Taxonomy to insure that some of the test items are at the higher cognitive levels. (One of the shortcomings of standardized tests is that most test items are aimed

at knowledge, comprehension and application, the three lowest cognitive levels.) To attempt to validate the cognitive level of your test items above the knowledge level get the opinion of your colleagues.

Phrasing Your Questions

The incorrect options of a multiple-choice test item are called foils or distractors. To the less able student they should represent a reasonable and attractive answer; to the more able student they should appear less attractive than the correct option. No one is in a better position to write good foils than the teacher who has taught the course. When you look at a particular problem you can predict, on the basis of your past experience, those answers that are likely to arise from incorrect reasoning. You should collect illustrative foils throughout the year by noting students' incorrect answers to your formative tests. Since it is unlikely that anyone can predict all of the incorrect answers to a given question, use "none of these" or "none of the above" as an option on some test items. (However, if this option is used, it should be the keyed option 20% of the time for a five-option multiple-choice item.)

Directions

Since you plan to revise your test on an annual basis, you will want students to answer all items. This will give you some insight into the quality of the foils you have developed for each item. To prevent students from omitting questions, the directions might include the following information: "You will receive your best score by answering every question because your score is the number of correct answers you mark. If a question seems to be too difficult, make the most careful guess you can, rather than waste time puzzling over it." Research indicates that better students tend to improve their score when guessing, while poorer students do not.

Revising and Improving Your Test

Once the test has been constructed and administered, you should subject it to item analysis, probably the most important aspect of test development. To begin, designate the best 27% of the tests as the "high group" and the poorest 27% of the tests as the "low group." A tabulation form like the one below could be helpful.

Item Number	Group Level	Option Selected A	B	C	D	E	Omit	P_H	P_L	D	P
	H										
	L										

P_H represents the proportion of the examinees in the high group who answered a given test item correctly and P_L represents the proportion of

examinees in the low group who answered the given item correctly. An index of the discriminating power of an item, $D = P_H - P_L$, is obtained by subtracting the two proportions as shown; and an index of the difficulty of an item, $P = (P_H + P_L)/2$, is obtained by averaging the two proportions. P is actually the proportion of the total number of examinees, in the combined two groups, who answered the given item correctly.

The examples that follow are taken from the item analysis of a 55-item multiple choice test administered to 148 students. Here N = 148 and n = .27(148) ≐ 40. ("n" is the number in the high group or the low group.) An "ideal item" (#32) under item analysis with choice C as the keyed option:

Item Number	Group Level	Option Selected A	B	C	D	E	Omit	P_H	P_L	D	P
32	H	0	0	40	0	0	0	1.0		.80	.60
	L	8	8	8	8	8	0		.20		

The index of discrimination is high, the foils are equally attractive, and the difficulty index is near .5.

Question #23: $\frac{(3x)^3}{3x^3} =$

A. 1 B. 3x C. 9x D. 9 E. 27

Item Analysis

Item Number	Group Level	Option Selected A	B	C	D	E	Omit	P_H	P_L	D	P
23	H	0	1	1	38	0	0	.95		.88	.51
	L	26	3	3	3	2	3		.075		

The item analysis of item 23 is probably more typical of what actually happens with "good" items. The discrimination index is high, the index of difficulty is near 50%, but the foils are not all equally attractive.

Question #15: The statement "m is 3 less than n" may be written as:

A. m − 3 = n B. n − 3 = m C. m = n + 3
D. m − n = 3 E. n = 3 − m

Item Analysis

Item Number	Group Level	Option Selected A	B	C	D	E	Omit	P_H	P_L	D	P
15	H	2	37	1	0	0	0	.93		.18	.84
	L	5	30	4	1	0	0		.75		

Item 15 is also somewhat typical for an item which should occur early in a test. It has a marginal discrimination index, but the difficulty index indicates that it is a relatively easy item. It is good practice, psychologically, to place some less-difficult items at the beginning of the test.

Question #54: An automobile travels $\frac{a}{6}$ ft. in r seconds. How many yards will it travel in 3 minutes?

A. $\frac{30a}{r}$ B. $\frac{a}{360r}$ C. $\frac{10a}{r}$ D. $\frac{a}{1080r}$ E. $\frac{30r}{a}$

Item Analysis

Item Number	Group Level	Option Selected A	B	C	D	E	Omit	P_H	P_L	D	P
54	H	10	5	3	2	3	17	.075		.05	.05
	L	2	8	1	1	3	25		.025		

Item 54 is an example of an item which ought to be omitted or revised. The item is too difficult in its present form and the low discrimination index indicates that the item tends to lower the reliability of the test as a whole.

Question #21, below, is an example of a text item which could exhibit a negative discrimination index — something to be avoided at all costs.

21. If $x = \frac{a^3}{(a-4)^2}$, then $\log x =$

A. $\frac{\log a^3}{\log(a-4)^2}$ B. $\log a^3 - \log(a-4)^2$ C. $\log a^3 + \log(a-4)^2$

D. $3 \log a - 2\log(a-4)$ E. $3\log a - 2\log a + 4$

Options B and D are both correct, depending upon the examinee's previous experience. While option D might be considered a "better" answer, without better instructions, the examinee is likely to choose the first option which is correct. If "best answer" is wanted, "best answer" should be specified and the examinee should be able to choose without doubt.

In general, once the discrimination index and the difficulty index have been obtained, one can begin revising the test. Items which exhibit little discriminating power ($D < .2$) should be carefully scrutinized to determine how they can be improved, unless these items are among the "easy" items which normally occur at the beginning of the test. Items which indicate a considerable level of difficulty ($P < .20$) should also be revised, omitted, or replaced if the discrimination index is also low. The difficulty index also provides a means for ordering test items for the next form of the

test. In general, the discriminating power of each item contributes to the reliability of the test as a whole. For maximum discrimination, the average difficulty index P should be 0.5.

When the revised test is administered, a new item analysis should be performed to learn to what extent the items have been improved. Questionable items should be fewer and such items should be revised. Each time the test is given, you should determine a measure of reliability. A reliability coefficient of 0.9 or higher is desirable. Some software packages have provisions for item analysis and determining reliability coefficients.

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MATHEMATICS EDUCATION MONTH
National Logo Contest

All students in grades K-12 are encouraged to participate

Guidelines:

- Designs for use on posters and buttons must be related to mathematics.
- All entries must be originals on paper no larger than 21 cm x 30 cm and become the property of NCTM. Copies or adaptations of other artists' works, cartoons, or logos will not be considered.
- Entries will be judged on the originality of the idea or concept rather than artistic ability.
- Computer-generated entries must be accompanied by an unprotected disk containing the program.
- Entries from any local, state, and provincial logo contests may also be submitted in the national contest.
- Each entrant's name, grade, and school address must be typed or printed legibly on the back of the entry.
- Entries must be mailed to NCTM by a classroom teacher or mathematics supervisor whose name, address, telephone number, and signature, plus member number for NCTM members, appear on the back of the entry as verification of originality.
- A $3 submission fee must accompany each entry.
- All entries and fees must be received by 30 April 1986.

Prizes:

All students submitting entries to NCTM will receive a certificate of participation.

One student winner from each state, the District of Columbia, the U.S. Territories, each Canadian province, and one international winner will receive a solar-powered calculator.

One national winner will receive a microcomputer system, and his or her logo will be used in conjunction with Mathematics Education Month 1987.

Bonus Awards:

All NCTM members forwarding entries that are chosen as national winners will receive a complimentary one-year extension of their NCTM membership services if their NCTM member number was included on the back of the entry sheet at the time of submission.

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REGIONAL COMPUTER RESOURCE CENTERS: OPPORTUNITIES FOR EXCELLENCE

by
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Computers have been available to teachers since the 1960s. However, because of high costs associated with mainframe computing, difficulty associated with batch processing and time sharing, and the lack of useful educational programs, these machines were generally not used in typical classrooms. When machines were available, they were frequently preempted for administrative uses.

With the public appearance of microcomputers in 1977, their potential for use as a classroom tool and resource took a quantum leap forward. These new computers were small, portable, and reliable. Best of all, they were affordable. Unfortunately, good software was not immediately available nor was there an adequate number of teachers trained in the classroom use of microcomputers. As schools began to purchase this new technology, professional debates regarding the use and implementation of microcomputers began to occur, and questions generally outnumbered answers.

The term, "computer literacy," was coined and was quickly accepted by many educators as something that all schools should be teaching; however, the professional literature clearly shows that there is widespread disagreement regarding the definition of computer literacy. Luehrmann (1981) argues that literacy in language means the ability to read and write; literacy in mathematics means the ability to do mathematics; likewise, computer literacy means the ability to do computing. In contrast, Johnson et al. (1980) broaden the scope of computer literacy to include applications, impact, software and data processing, hardware, and affect. Lee The (1982) suggests that rather than discussing computer literacy which is often focused on the computer as an end in itself, what is needed is computing literacy which he defines as the knowledge and ability to use computing systems as tools for the enrichment of our personal and professional lives. This would include a combination of skills embodied in applications of software deriving from practical experience and resulting in the use of the computer as a machine, tool, and a creative instrument.

Anderson (1981), Gress (1981), and Eisele (1980) as well as a seemingly endless list of others, have written about the concept of computer literacy and their view of what should be included in a good curriculum. There is

no doubt that disagreement exists over what constitutes computer literacy, but this does *not* suggest that schools should abandon their efforts to reach a local decision regarding which form of computer education would be best for the students in that particular district.

In contrast to the continuing debate over the definition of computer literacy and how computers should be used in schools, there is generally more agreement regarding the manner in which computers can be used. Mandell (1982) surveyed the literature and reports that four methods for using the computer in the classroom can be easily identified. They include using the computer to: (1) individualize instruction through interactive learning experiences at appropriate levels of difficulty; (2) provide tutorial assistance by dispensing information in a novel manner; (3) develop program writing skills which can then be used to enhance the learning of other material; and (4) enable evaluation of learning with immediate feedback and reinforcement. In addition the literature describes the record-keeping and data management applications for the classroom teacher and the school administration.

The use of the computer in an educational environment has frequently been viewed as being either computer assisted instruction (CAI) or computer managed instruction (CMI). In the former case we can include applications in which a student engages in dialogue with a computer program to achieve a well defined and measurable understanding or skill (Lower et al., 1979). In the case of CMI we would include software that was useful to the teacher in his/her record-keeping and materials generation tasks. In both cases there are many subcategories of programs from which teachers must make informed decisions.

MICROCOMPUTER SURVEY RESULTS

Findings

A survey of computer usage in American schools was conducted early in 1983 by Johns Hopkins University's Center for Social Organization of Schools. The project director, Henry Jay Becker, reported several findings (1983), many of which can be translated into concerns:

> *Most of the software being marketed for schools is targeted at the elementary school level and is designed around the premise that microcomputers can be a cost-effective means of increasing the rate at which students learn rules of arithmetic computation and proper English language usage.*
>
> *Secondary schools remain the largest precollege users of microcomputers, and that the overwhelming emphasis in secondary schools is on teaching students about computers and how to program them using the BASIC language.*

These two findings, at best, indicate a need for better communication between the producers and the consumers of the software. Further, an initial lack of good software was a reason for many early users to become disenchanted with the value of microcomputers in the classroom.

Fifty-three percent of all schools in the United States had at least one microcomputer available for instructional use.

Secondary schools are more likely to have a micro than elementary schools. Eighty-five percent of the high schools had at least one microcomputer, but only forty-two percent of the elementary schools could make that claim.

Although it is encouraging to see microcomputers being purchased for use in the schools, there is still an enormous shortage of computers available for general classroom use.

Drill/practice and programming instruction were the two primary uses of the microcomputers.

Although it is encouraging to see the microcomputers being used, it is disconcerting to note that the full power and capabilities of the microcomputer are not being utilized. Simulations, word processing, teacher test construction, student papers, recreational use, and tutoring are only infrequently being done with the aid of a computer. None of those tasks is done by 75% of the teachers reporting to be "regular" or "intensive" users of the microcomputer.

In schools that obtained a microcomputer before July 1981, nearly two-thirds of the secondary and one-third of the elementary teachers who said that the micro was originally viewed as a tool now view it as a resource. In contrast, only about 10 percent of those who initially viewed it as a resource now consider it to be primarily a tool.

Given the potential for using the microcomputer to teach subject concepts in geometry, physics, English, and in other areas, it is alarming to find that the experienced teachers view the micro as a resource to learn more about computers rather than as a tool to help them teach content skills in traditional disciplines. This appears to be true even for teachers who initially purchased the computer as a tool. Becker (1983) suggests that this situation may have resulted from "pioneer" schools becoming disenchanted with the drill-and-practice software available at the time and have not yet ventured back to examine more recent software products.

In about half of the schools with micros, only one or two teachers, at most, are regular users.

In a few schools, primarily elementary schools, no teacher regularly uses the school-owned microcomputer with their students.

An interesting dilemma is present in some schools that have only a small number of microcomputers; administrators wishing to ensure that the tax dollars are being spent wisely require that future micro purchases are contingent on more use of the existing computers. However, teachers

argue that more machines are necessary before any of them can be used more effectively.

> *As elementary schools get more microcomputers, they give access to more students.*
>
> *As secondary schools get more microcomputers, they give longer access to the same number of students.*

Although lengthy discussion could accompany the above two findings, it appears to be safe to state that, in general, neither the elementary schools nor the secondary schools have yet acquired a sufficient number of micros to allow all of the students sufficient time to adequately utilize the computers for the tasks at hand.

Problems

Findings such as the ones listed above suggest some very basic problems which need immediate attention. These problems include:

> *There are not enough microcomputers in the schools to provide the interested teachers with sufficient flexibility to effectively and conveniently utilize the machines with all of their students.*
>
> *Some of the existing software, particularly that produced a few years ago, is not useful to the teachers.*
>
> *Good software that exists is not easily found by the teachers.*
>
> *Many curriculum guides emphasize the use of computers in drill-and-practice exercises at the expense of more sophisticated applications such as simulations; hence, the full capability of the computers is not being used.*
>
> *Many schools are neglecting the potential of the computer in the teaching of subjects such as geometry, physics, and English.*
>
> *Many teachers are enthusiastic about the potential of using microcomputers; however, a lack of in-service training exists resulting in large numbers of teachers who feel inadequate or insecure using the computers in the classroom.*

Opportunity for Solutions

The problems outlined above are soluble; however, the solutions generally require time and money. Given the depressed financial condition in many of the school districts throughout Pennsylvania and the monumental nature of microcomputer education, it was evident to a growing number of people that if the above problems were to be addressed in any significant manner in Pennsylvania, it would require a coordinated effort throughout the Commonwealth. Further, this effort would require both fiscal support and a state-wide organization with strong leadership.

There appeared to be little doubt that educators in Pennsylvania needed and wanted assistance in the implementation of computers into

their classrooms. Several people felt that one approach to the solution of the problems facing educators would be to establish several coordinated resource centers throughout the Commonwealth.

This network approach appealed to Mr. Ken Reeher, Executive Director of PHEAA, and his agency agreed to work with the Pennsylvania Science Teachers Association through its president, Dr. Ken Mechling, and Representative David R. Wright to establish a plan to assist educators in their use and understanding of computer information technology. A multi-faceted plan called the Pennsylvania Science Teacher Education Program (PA STEP) was developed and PHEAA agreed to provide funding.

The PA STEP program includes workshops for school administrators, science instruction for elementary teachers, college tuition loan forgiveness plans for highly qualified people entering mathematics or science teaching in Pennsylvania, and a program called CORES (Computer Orientation in Reshaping Education in Science).

The CORES program was designed for mathematics and science teachers who wanted to improve their understanding of how microcomputers could be used to improve classroom instruction. This program began in September of 1983 through the offering of free graduate-level instruction in microcomputer education to secondary mathematics and science teachers throughout Pennsylvania. CORES was an instant success; teachers who completed the course recommended it highly to others, and waiting lists to get into the program grew rapidly at all six centers offering the CORES program.

Because of the large number of applicants to the CORES program and because of the equally important needs of other teachers in grades K-12, PHEAA saw a need to extend the boundaries of microcomputer education for teachers. With leadership by Representative Wright and PA STEP Director, Ken Mechling, House Bill 1898 was drafted. This bill was in fact an extension of the CORES program and provides for a comprehensive approach to microcomputer education for all teachers in Pennsylvania.

In July of 1984 the General Assembly of Pennsylvania passed House Bill 1898. This act became known as the Information Technology Education for the Commonwealth (ITEC) Act, and its general purpose is to raise the floor of microcomputer literacy among Pennsylvania educators. As stated in the ITEC summary report for 1984-1985 (Mechling, 1985), the law intends to (1) provide teacher training in computer information technology, (2) help design computer-oriented elementary and secondary curricula, (3) assist teachers and school administrators in the evaluation of educational computer software, (4) administer and fund individual school's proposals to acquire and upgrade computer equipment and implement computer-oriented instruction, (5) assist in the coordination of the purchase of microcomputer hardware and software for Pennsyl-

vania schools and (6) loan equipment and computer software to non-public schools.

Fourteen statewide Regional Computer Resource Centers (RCRC's) and four outreach sites have been established to provide a wide variety of support to the educators in Pennsylvania. Figure 1 shows the location for each of the fourteen RCRC's. Figure 2 lists the contact person for each RCRC. Administration of these centers is being coordinated by the Teacher Education Center at Clarion University and by the Pennsylvania Higher Education Assistance Agency (PHEAA).

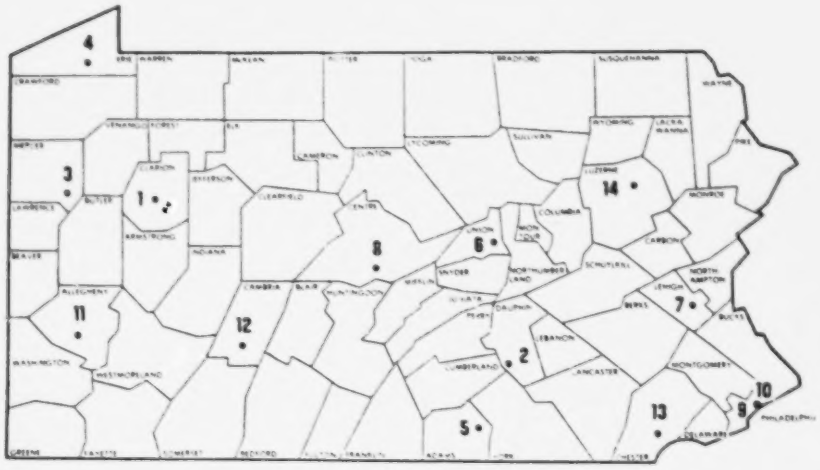

Figure 1

DESCRIPTION OF OPPORTUNITIES AND SOLUTIONS

Graduate-Level Courses

A major component of both the PA STEP program and the ITEC Program is the availability of tuition-free graduate-level courses at each Center. Since a fundamental philosophy of the program is to raise the floor of microcomputer literacy among Pennsylvania educators, most of the courses offered are introductory in their nature and are designed with the microcomputer novice in mind. During the 1985 spring semester, 31 ITEC courses were offered statewide with over 1,000 elementary and secondary teachers from all disciplines being selected for admission into these introductory level classes. To facilitate instruction, most of the centers offer a section of the introductory-level course that is oriented specifically to mathematics and science teachers. These sections are typically referred to as CORES (Computer Orientation in Reshaping Education in Science) courses. Non-mathematics and non-science teach-

WHEN AND WHERE ARE ITEC COURSES OFFERED?

ITEC courses are offered all year round at 14 Regional Computer Resource Centers (RCRC) throughout the Commonwealth. Refer to the map to find the RCRC nearest you, then contact the RCRC director and find out the exact dates for courses beginning soon. DO IT NOW and come on line!

Clarion University
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Clarion University
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Harrisburg Area Community College
Director Mr. Barry Barnhart
HACC
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2 Harrisburg, PA 17110
(717) 780-2569

Intermediate Unit No. 4 — Midwestern
Director Mr. Angelo Pezzuolo
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Intermediate Unit No. 5 — Northwest Tri-County
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Intermediate Unit No. 12 — Lincoln
Director Mr. John Asch
Shippensburg University
Shippensburg, PA 17257
5 (717) 532-1673

Intermediate Unit No. 16 — Central Susquehanna
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Figure 2

ers from grades K-12 usually will enroll in the ITEC classes. The objectives for all of these introductory-level courses include the teacher's ability to:

(1) evaluate, select, and recommend for purchase microcomputer hardware, software and courseware to meet their classroom and laboratory needs.
(2) use microcomputers in their own classroom and laboratories to teach concepts/principles and to meet other academic and administrative goals.
(3) demonstrate entry-level competence in the BASIC language.
(4) recognize philosophical, ethical, and legal implications for microcomputer use in an educational environment.

(5) understand much of the common computer jargon in frequent use today.

During the 1985-1986 academic year an intermediate level course was introduced. This course (referred to as CORES II for mathematics or science teachers and ITEC II for all other teachers) assumes knowledge and skills equivalent to those developed in the introductory level course, and continues to provide educators with an opportunity to examine ways to better utilize computer technology in the classroom. The objectives in this course provide the mathematics teacher with an opportunity to:

(1) demonstrate the use of integrated management software to handle record keeping, reporting, test generation, student and parent notices, etc.
(2) evaluate, select, and integrate courseware with curriculum in the classroom learning environment.
(3) utilize communication systems to access, retrieve, and store information.
(4) produce instructional resources for use in the class.
(5) use documentation and manuals for software, hardware, and peripheral utilization, installation, and maintenance.
(6) use microcomputers for direct laboratory measurement.

Hardware/Software Grants

Another important feature of the ITEC Program is the opportunity for public school districts within Pennsylvania to apply for hardware/software grants of up to $40,000 per year. (Non-public schools are eligible to request a loan of computer equipment and/or software.) In June of 1985 there were 55 awards totaling over $1,960,000. Proposals were scored on the basis of quality, state aid ratio, and student/computer ratio. It is anticipated that these awards will be repeated during each of the four years of the ITEC Program; however, the guidelines and dollar values are expected to change from year to year.

Software Library

Each RCRC has a very large collection of educational software. This collection is balanced among all of the grades and disciplines; however, most centers have an especially good collection of mathematics-related programs. The purpose of this collection is to provide you with an opportunity to preview and review software prior to making a purchasing decision. Most of the centers have a good collection of print materials such as the MicroSIFT Courseware Evaluation or the Micro-Courseware Pro/File and Evaluation by EPIE (Education Products Information Exchange) and the Consumers Union. Materials such as these are invaluable in a software review process. With the help of these reference materials and the guidance of one of the RCRC staff members, you can quickly develop a feeling for the quality of the exemplary software now on the market.

You may choose to review integrated management software that could be useful to you by reducing the time that you spend doing routine "bookkeeping." AppleWorks is one program that many teachers have found to be especially useful for writing exams, preparing handouts, and making worksheets. Its integrated spreadsheet has potential as a means for mathematics teachers to customize a class gradebook, and its database function is an easy way to keep track of classroom materials or to organize data about your students or curriculum.

Or, you may ask for a collection of teacher favorites such as Print Shop, Crossword Magic, Word Search, and Newsroom. Using programs such as these, the mathematics teacher can quickly and easily prepare professional-looking crossword puzzles and other handouts appropriate for use in your mathematics classes.

You may decide to review some of the excellent mathematics programs such as Rocky's Boots or Green Globs, a graphing activity in a game format. If you choose to review mathematics software and you want to see the power of the computer being utilized, be certain to look beyond programs that do only drill and practice activities.

Hardware

Each of the fourteen Regional Computer Resource Centers has an extensive array of hardware. Although this hardware varies from center to center, its purpose for being there does not. That is, you are invited into the RCRC to compare, contrast, and to use these pieces of equipment. In addition to a variety of computers, monitors, drives, and printers, you may find plotters that would be of special interest to mathematics teachers. You may also find optical scanning devices that are connected to a computer, digitizers, speech synthesizers, large screen projection devices, interactive laser equipment, graphics tablets, networks, and modems capable of accessing electronic bulletin boards such as Compu-Serve. If you teach a computer literacy course or a programming course, a visit to one or more of the centers would be very worth your time. Many districts allow these visits in lieu of an in-service day.

Workshops and/or Special Programs

Depending on the availability of RCRC personnel, most of the centers are willing to entertain special requests for workshops or programs that may be unique to your department or school. One idea would be to arrange for your entire mathematics department faculty to spend an in-service day at an RCRC. While at the center you may elect to spend your time reviewing mathematics software or learning to use a specialized piece of equipment. The day can be as structured or as informal as you desire.

AN AGENDA FOR ACTION

Given your probable high level of interest in computer education

(based on your endurance for reading this far into the article) and given the availability of, and support from, the Regional Computer Resource Centers, you are almost ready to implement some new ideas and software into your curriculum. But first, you should believe that the use of computers in your classroom is useful and not just another technological fad. Finally you will need a plan of action, and then you will be ready to make additional progress in improving your efficiency and updating your methods of instruction.

Luerhmann (1982) provides some assurance regarding the "staying power" of computers when he states that the computer is unique among information technologies in that it permits intelligent interaction with the learner. All previous technologies have been one-way paths for distributing facts and ideas. As a result of this and because of other factors, the microcomputer has tremendous potential for long-term use in the classroom.

The National Council of Teachers of Mathematics has taken a leadership role and it has developed and published *An Agenda For Action*. This document emphasizes that its recommendations are not an end, but rather a beginning. The recommendations represent an agenda for a decade of action, and it is significant that the NCTM plan recognizes the potential of computers in classroom education. Of the eight recommendations made by the NCTM, two are very closely related to the use of computers in education. The Council recommended that problem solving be the focus of school mathematics in the 1980s and that mathematics programs take full advantage of the power of calculators and computers at all grade levels. In an elaboration of the latter recommendation, the council listed twelve specific actions which should be taken to accomplish this recommendation. Included in the actions were the following:

(1) All students should have access to calculators and increasingly to computers throughout their school mathematics program.
(2) The use of electronic tools such as calculators and computers should be integrated into the core mathematics curriculum.
(3) Curriculum materials that integrate and require the use of the calculator and computer in diverse and imaginative ways should be developed and made available.
(4) All mathematics teachers should acquire computer literacy either through preservice programs or through in-service programs funded by school districts in order to deal with the impact of computers on their own lives and to keep pace with the inevitable sophistication their students will achieve.

You are familiar with the historic developments leading to the widespread availability of microcomputers in the classrooms. You have read about some of the issues surrounding the implementation of the computer. You are familiar with the National Council of Teachers of Mathematics *Agenda for Action*. You are cognizant of the overwhelming work

yet to be done. You now know of the wide variety of support available to you from the state-wide network of Regional Computer Resource Centers. If you haven't taken steps to continue your own professional growth or to encourage your peers, what are you waiting for?

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National Council of Teachers of Mathematics

Proclamation

Whereas, mathematical literacy is essential for citizens to function effectively in society; and,

Whereas, mathematics is used every day - in the home, on the job, and for recreational activities; and,

Whereas, the language and processes of mathematics are basic to other areas of the school curriculum; and,

Whereas, expanding technology demands increased mathematical competence; and,

Whereas, relevant community and school activities can generate interest in mathematics;

*Now, **therefore**,* I, F. Joe Crosswhite, president of the National Council of Teachers of Mathematics, do hereby proclaim the month of April 1986 as

Mathematics Education Month

to be observed in schools and communities in recognition of the importance of mathematics.

In witness whereof, I have hereunto set my hand and caused the Corporate Seal of the National Council of Teachers of Mathematics to be affixed on this 1st day of January 1986.

SEAL

F. Joe Crosswhite

President

EVALUATION OF MICROCOMPUTER COURSEWARE

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Computer assisted instruction (CAI) provides most of the instructional characteristics of an optimum tutor: presents information, asks questions, provides wrong and right answer feedback, records performance, and facilitates curriculum continuity for computer-based instruction. Courseware, as used in this article, includes the microcomputer disc or program, the documentation manual, the teacher's manual and the supporting curriculum materials needed in CAI. Teachers and curriculum supervisors need guidelines for an evaluation process which will increase their chances of choosing high-quality instructional courseware. By using the following guidelines for the evaluation process, these professionals can become well-informed consumers.

Assumptions Underlying Guidelines

Before using the guidelines, the user should keep in mind four assumptions made in the development of the checklists and the recommendations. *First*, the role of a set of guidelines is to focus attention upon key issues critical to making an informed judgment. *Second*, each user will assign more weight to some items than others based upon the instructional uses he or she has in mind. Consequently, some items may be judged not relevant to a particular use. Hence, a decision about the quality of a piece of courseware should be guided by the overall rating for each of the sections rather than any individual item. *Third*, a professional user may want to add items or sections to meet a purpose not anticipated by the author. They should do so providing the rationale for such additions and the checklist of items to be used to reach an overall rating is provided. *Fourth*, the assessment process assumes the user will choose instructional courseware for evaluation that he/she believes has the potential for meeting the needs of a specified audience before applying the guidelines.

The guidelines are organized into seven sections. Each section includes the background and rationale which identifies issues and state-of-the-art knowledge critical to providing high-quality CAI, a recommendation translated into a checklist of evaluation criteria, and an overall rating scale and space to note the particular strengths and weaknesses related to a particular section. The guidelines were developed as part of the Penn State/Keystone Area School District Teacher Corps Project. (See Nelson &

Trueblood, 1984; Trueblood & Flanagan, 1983-84) The reason for placing the evaluation criteria in each checklist is that they summarize what research and our experience has shown to be the key characteristics of effective CAI. It is important, therefore, to consider all of the criteria in a checklist for a section and the related recommendation before giving the courseware being evaluated an overall rating.

General Directions

First, assemble all of the computer programs, the documentation and teacher's manuals, and the type of microcomputer needed to review the programs you wish to evaluate. Then complete the remaining five steps in the review-comparison-assessment process. These steps are summarized to provide an overview of the assessment procedure that should be followed.

STEP 1. List in writing the reasons for evaluating the courseware you have assembled. The results of this step, the standards and the ratings from the guidelines that follow, will provide you with the data needed to decide which courseware will best meet your needs. So, include:

A. The instructional objectives the courseware will meet.
B. The type/level of students who will use the programs.
C. The type of program(s) you think will meet the objectives you've identified. Also, record the price range within which you have to work.
D. The way in which courseware will fit into the curriculum and the instructional process now used in the classrooms. For example, is the program to be supplementary to, or will it replace, a unit, module or course that is now in place?

STEP 2. Next, execute each of the programs as a "poor" or unsuccessful student would. Test all wrong answers to see how extensively the program handles these responses. Also, test to see how the program handles typing errors, incorrect input (e.g. syntax and spelling errors), and errors in following directions. Record the time it would take such a student to complete the program. The purpose for this step is to determine the extent to which the program can meet the needs of the students who need extensive instruction.

STEP 3. Load and run each of the programs as "good" or advanced students would. Check all of the correct answers. Also extend and check the program's capacity as a creative user would. The purpose of this step is to test the comprehensiveness and cleverness of the programmer. Also, record the amount of time it would take such a student to complete the program.

STEP 4. Next, read and complete the rating scales provided in each section of the Guidelines for Evaluation. Record your rating for each

program in different colored pencil or pen. This color coding will help you make a comparison in the next step. Note the strengths and weaknesses of each piece of courseware under the appropriate rating scales. Lastly, check the vendor's claims. Record those that are accurate and those that are not accurate.

STEP 5. Using the data you've assembled, rate the programs in terms of how they will meet the student needs identified in Step 1. You may also want to justify the programs in terms of cost, instructional time and usefulness to the teacher.

THE GUIDELINES

SECTION 1. Quality of Educational Objectives

Students are more motivated to perform well whenever the objectives are at the student's instructional levels rather than their frustration levels. They are more self-directed whenever they know what they will be able to do after instruction. Teachers can more easily integrate microcomputer-based instruction into their ongoing program whenever they know what level of outcomes are intended by the authors of a program.

Recommendation #1. Educational objectives and performance criteria should be available to both teachers and students in a form that is clear and concise. The level of the objectives should be appropriate for the intended audience.

Assessment Procedure. Locate the objectives in the documentation manual, the teacher's manual, and/or record those presented on the screen, then check the objectives against the following criteria.

____ The objectives are worded in clear and concise form.
____ The type and level of instructional objectives are correctly identified.
____ The level of objectives is appropriate for the age/level specified.
____ The objectives are made available to teachers.
____ The objectives are made available to the students.
____ The objectives are at the intended audience's instructional level.

Using the following scale, rate the overall quality of the program's educational objectives.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 2. Adequacy of Instructional Design

To determine to what extent a program adheres to the fundamental principles of good instructional design requires that four characteristics

be evaluated. These characteristics include determining whether or not the authors have: (1) placed standard instructional events in the correct order, (2) made appropriate choices regarding instructional strategies, (3) chosen a logical programming format, and (4) provided students with quality feedback and corrective design to improve their performance.

SECTION 2A. Flow and Sequence of Instruction

The flow of effective instruction is characterized by standard episodes. These episodes should occur in the following order: checking for prerequisite knowledge, establishing the context for new learning, presenting the new learning, checking for mastery or understanding, and teaching and checking for transfer or retention. These episodes take on different forms depending upon the creativity of the program's author and the nature of the instructional objectives to be achieved. Not all of the episodes must take place on the machine. The materials needed to provide an episode off the machine can be provided via the teacher's manual or supplementary materials that accompany the program. In either case what is provided by the software and what is to be provided by using the teacher's manual should be considered when rating the flow and sequence of instruction.

Recommendation #2. The instruction provided on the machine and/or the teacher's manual should at a minimum provide ways for checking prerequisite knowledge, establishing the context for new learning, providing new learning, checking for mastery or understanding, and teaching for transfer.

Assessment Procedure. Using the following checklist, record whether or not the standard episodes are provided in the correct order by the program and/or teacher's manual.

Standard Episode Checklist

____ Recall of prerequisite knowledge is evaluated or reviewed before presenting new material.
____ Advanced organizers or orientation to new learning is presented prior to such learning.
____ New learning is presented in an appropriate context with the amount of guidance appropriate to the instructional strategy used.
____ Mastering or understanding of new learning is evaluated.
____ Practice exercises are provided to enhance transfer and retention of new learning.

Using the following scale, rate the flow and sequence of instruction:

Inappropriate						Appropriate
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 2B. Match between Programming Format and Program Objectives.

Different programming formats are selected by authors depending upon the type of objectives to be achieved. For example, teaching relationships and solving problems are accomplished through simulations. Simulations are most appropriate when actual experience is too costly, unsafe, or too time consuming. Branched programs provide remedial possibilities and therefore are most appropriate when a diagnostic approach to teaching skills or concepts is desired. The linear drill and practice format is most appropriate whenever repeated use of previously learned content is desired. In some cases a combination of formats should be used.

Recommendation #3. The type of programming format should match the level and type of instructional objective(s) to be achieved.

Assessment Procedure. Use the following steps and the results from Section 1, Quality of Educational Objectives, to detemine what the match is between the program objectives and the type of instructional objectives.

Step 1. Use the following checklist and identify the types of objectives.

____ Recall of previous knowledge.
____ Understanding rules, relationships, concepts.
____ Problem Solving (applying rules, concepts, relationships).
____ Locating and using information from a data base.
____ Mathematical Computation.
____ Other (specify).

Step 2. Use your notes from reading the documentation and teacher's manual, working through the program as a "good" student and a "poor" student, and the information in the following table to identify the type of program formats. Use the characteristics in the following table and determine which programming format(s) is or are used.

Standard Programming Formats	Key Characteristics
____ Branching Tutorial	Students get different information, questions, feedback and alternative sequences of instructional correctives depending upon their responses.
____ Linear Tutorial	All students get same information questions, feedback, and sequence of instruction regardless of their responses.
____ Simulation/Problem	Information is provided about a system. Student selects options to find out what

	impact it has upon the system. Computer shows how system has changed based upon student choices.
____ Instructional Game	Information is presented, student A responds, computer responds using algorithm unknown to student or another student B responds to counter student A's move.
____ Information Retrieval	Student uses coded data or description to get information about specific topic or key questions from a data bank.
____ Drill and Practice	Students supply facts, terms, definitions, etc., requested and they receive feedback about the correctness of their answers.
____ Other (specify)	Provide description.

Step 3. Compare the results from Steps 1 & 2 and determine to what extent the programming formats match the types of program objectives.

Rate the match among Programming Formats and the program objectives:

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 2C. Correspondence Between Instructional Strategies and Program Objectives.

Instructional strategies are judged effective whenever they enable students to meet the program's objectives. Therefore, it is important to identify the instructional strategies used in the program and to determine to what degree the student performances required by the strategies correspond to those of the program's objectives.

Recommendation #4. There should be a high correspondence between the instructional strategies used in a program and the level of performance stated in the program's objectives.

Assessment Procedure. Use the following steps and the results from Section #1 and determine how well the instructional strategies match the program's instructional objectives.

Step 1. Use the following chart and identify the types of instructional strategies used in the program.

Strategies	Description of Strategies
____ Drill and practice	Students supply facts, terms, definitions, etc.
____ Concept attainment	Students identify examples and non-examples of a concept.
____ Rule — example	Students are given a rule; rule is applied to an example.
____ Example — rule	Student is given examples; student identifies a rule; rule is applied.
____ Problem Solving	Students are given an unfamiliar situation; questions are asked; student produces a solution; solutions are evaluated.
____ Relationship among variables	Students respond to a system by making decisions; deduce a relationship among variables based upon the system's response to their decision; test the relationship in simulation format.
____ Information Review	Students read verbal text, graphic displays, etc.; students tested on meaning of content in material presented.
____ Information Retrieval	Students use code words to gain access to information; information given on screen or printout.
____ Guided Examination	Students are presented with information; answer convergent questions and select or state a rule or answer divergent questions and select and evaluate alternative answers.
____ Gaming	Students use previous knowledge to beat the computer or human opponent in a game format.

Step 2. Compare the results from Step 1 with the program's instructional objectives identified previously in Section 1. Decide to what extent the instructional strategies match the type and level of performance stated in the program's objectives.

Rate the correspondence between the instructional strategies and the program's objectives.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 2D. Quality of Feedback

A student's knowledge concerning the correctness or incorrectness of his/her performance has a powerful influence upon his/her present and future learning. The specificity and immediacy of feedback affect both the quality of students' performance and their interest in learning. When students know how well they are doing, what needs to be improved and what to do to bring it about, they are more motivated to try and improve their performance.

Recommendation #5. The type of feedback should be specific and tailored to fit the student's response. Feedback should give students immediate knowledge about the degree of correctness of their answers. The tone of the feedback should be student supportive.

Assessment Procedure. After working through the program, use the checklists below and record the type and specificity of feedback. Also note whether the general tone of the feedback is student supportive.

Feedback types	Examples: tone & specificity of feedback.
____ Correctness only	words, audio tones, animation indicate answer are right/wrong; correct/incorrect; etc.
____ Positive reinforcement	words indicate nice job; excellent work; good; tones or animation used to supplement such phases.
____ Negative reinforcement	poor response; klutz; well ok but; no, let's try again; violent animation and/or audio tones indicate answers are incorrect.
____ Elaborated corrective	very good, you're right because . . .; no, x is not correct because . . .; try again and see if you can correct x; when answer is not correct initial instruction is repeated or remedial instruction is provided.

Rate the overall quality of the feedback.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 3. Gaining and Sustaining Attention

Students spend more time on learning tasks and learn more rapidly whenever a variety of motivation techniques are used throughout the program. In addition, programs that use strategies that get students ready to learn (e.g. stimulating recall of prior knowledge) are more effective than those that do not use such strategies.

Recommendation #6. A variety of attention-getting procedures that motivate students to interact with the computer program should be strategically located throughout the program. Appropriate readiness techniques that establish a learning set should precede the introduction of new concepts, skills or information.

Assessment Procedure. Use the checklist 3A and 3B below and record how many and what types of attention-getting and readiness techniques are used throughout the program. Decide whether or not their placement in the instructional program is appropriate and whether they are appropriate for the intended audience.

Section 3A
Attention-Getting Procedures

____ animation
____ flashing words or phrases
____ color/graphics
____ key questions
____ competition w/computer
____ audio messages
____ manipulation of screen displays
____ others (specify)

Section 3B
Orientation/Readiness Techniques

____ review of previous learning
____ analogies
____ review of prerequisite knowledge
____ overview of how new learning is organized
____ examples of how new learning can be used
____ discrepant events presented
____ others (specify)

3A. Rate the potential of the program for gaining and sustaining student attention.

Low						High
1	2	3	4	5	6	7

3B. Rate the program's use of Orientation/Readiness Techniques

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

_____

SECTION 4. Appropriateness, Currency and Correctness of Content

The program's content should be clearly outlined and specialized terminology carefully defined either in the teacher's manual or the program's documentation. Because of the current knowledge explosion, content changes rapidly and in our technological era can be rapidly changed and disseminated. Therefore, the currency and correctness of content should be checked so that students are provided with up-to-date information. Checking the sources for information and publication dates are two ways of assessing currency and validity of content. Comparing content presented by software against recognized textbooks, professional journals, electronic data bases and authorities in a particular field are other approaches to checking the correctness and currency of program content. The amount of content detail, social context, vocabulary level, and the sequence used to present the content are four factors that determine the appropriateness of content for particular age/developmental levels.

Recommendation #7. The program's content should be current, correct and clearly outlined in the teacher's manual or program's documentation. The content detail, social context, vocabulary level and sequence of presentation should be appropriate for the intended user's age and developmental level. The content should be free of cultural, ethnic and sex biases.

Assessment Procedure. Locate the content outline contained in the teacher's or documentation manual. Check the outline and the program's content against the following criteria.

____ Content is current/not dated.
____ The content comes from recognized authorities, professional organizations or data bases in the field.
____ Content of tests and text presented on the screen are free of content errors.
____ The basis for sequencing the content is specified or can be inferred from the program's content.
____ Scope and sequence of the content presented and the vocabulary are appropriate for the age/developmental level of the audience specified by the authors.
____ Content is free of cultural, ethnic and sex biases.
____ Content is grammatically correct, free of spelling errors and unneeded computer jargon.

Rate the appropriateness, currentness and correctness of the program's content.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 5. Quality of Evaluation Design

Instructional programs are effective whenever they help users achieve the programs' stated objectives. Programs that use tests with items keyed to the content and performance specified by the program's objectives are most likely to yield valid and reliable evaluation data. Quality programs provide for on-going assessment of student performance and make performance data available to teachers. Some programs also provide tests to be given off the machine before and after using the program. All test materials and related reliability and validity data should be described in the program's documentation and/or teacher's manual. This documentation usually includes a detailed description of the students used to test the program's effectiveness. The data source used to test the program's effectiveness is also identified in these manuals.

Recommendation #8. Items on tests should correspond closely to the content and student performance stated or implied by the program's objectives. Claims of test reliability and validity should be supported by data or references that provide such data. The evaluation design should be organized to provide teachers with student performance data at appropriate intervals throughout the program. The type of testing should be appropriate for the intended user's age/developmental level.

Assessment Procedure. Read the section(s) of the program documentation and teacher's manuals that describes how student performance is assessed. Check this information and the tests given on and off the machine against the following criteria.

____ Test items closely match the performance and content contained in the program's objectives.

____ Testing procedures are appropriate for the age/developmental level and background of the students used to evaluate the program.

____ Test reliability and validity descriptions and data indicate the testing procedure will yield valid and reliable information. Data is available to support the description.

____ Student assessment provided on the machine is placed at appropriate intervals.

____ Student performance records are a standard feature of the program or are made available through a special utility program.

____ Claims of program effectiveness are supported by appropriate student performance data or references that contain such data.

____ The student population used to test the program's effectiveness is described in sufficient detail to allow comparison with potential users.

Rate the quality of the program's evaluation design.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 6. Flexibility and Adaptability

Utility programs or features provide teachers with the flexibility, adaptability and time-saving devices required to use a program with a wide variety of students. Such utility features enable teachers to get student performance data, modify test items, add practice exercise and modify program content to meet the special needs of their students. They also provide overviews and suggest ways of using a program that help teachers make the best use of their planning time.

For obvious reasons utility features are usually built into a program so they are not visible to the students. These features should be described in detail in the documentation and/or teacher's manuals. In some cases separate disks are required in order to use the features. Some quality programs provide disks that tutor the teacher how to access these features and demonstrate what type of input and output are available. Some programs also illustrate such input and output in the teacher's manual.

Recommendation #9. The utility features for a program should provide teachers with the flexibility, adaptability and management capacity required to fit a program to the needs of a variety of students.

Assessment Procedure. Read the appropriate sections of the documentation and/or teacher's manual. Check the utility features described against the checklists below. Then use the trial runs previously suggested and see whether or not you can generate the displays or data indicated in the manuals. In cases where tutorial disks are available, simply follow the instructions and determine whether or not the results are readable and useful.

Management capacity — Use the following and check which management features (e.g. a menu) are available:

____ student records
____ data storage
____ specialized sub-routines
____ test item analysis
____ other (specify)

Program's modifiability — check which of the following the teacher can change without making major revisions to the program:

____ practice exercises
____ test items
____ learning activities
____ other (specify)

Teacher orientation and support features — Check which features are available.

____ detailed menus and course outlines
____ prompts that help the user modify the program
____ overview of the program and its purposes
____ supplementary materials and suggested uses
____ description of the relationship to other program disks

Rate the quality of the program's utility features below.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

SECTION 7. User Supportive

The content of this section influences the students' motivation to learn and their general attitude toward instruction they receive via microcomputers. This section draws attention to attributes that influence the amount of time it takes to complete a program (e.g. answer processing time, time required to load the program, etc.), the amount of control students have over the machine and the way the program processes equivalent answers, minor spelling errors and the number of commands needed to load and run the program.

Recommendation #10. A program should be free of system-type errors and technical requirements that prevent or impede users from responding to the instruction. Students, rather than the program, should have control over their rate of progress through the text presented on the screen. The written material should be at the intended user's independent reading level.

Assessment Procedure. Consider each of the following and how they might assist or prohibit students from moving through the program at a reasonable pace. Also consider both the age and microcomputer background that the intended user might need.

Checklist

____ simplicity and clarity of the load necessary to run the program are clear and simple to understand.
____ the program functions are clear and easy to access.
____ required knowledge needed to operate peripheral devices is minimal.
____ students have maximum control over text, menus and progress through program.
____ equivalent answers and simple spelling errors have been built in the answer processing.

____ text is at the reading level of the intended audience.
____ visuals and graphics are adequate.
____ audio messages are adequate.
____ number of system error messages is minimal.

Rate the user friendliness of the program.

Low						High
1	2	3	4	5	6	7

Note strengths/weaknesses below.

Conclusions

The long-term advantage of using evaluation guidelines such as those just presented is in establishing a file of evaluated courseware that can be quickly retrieved by teachers, administrators, supervisors and other professionals. A computer file would be the most flexible type of record since it can be accessed quickly, updated and stored in the least possible space. Such a file would also make courseware evaluations available to those who have established networks with other users.

Some courseware for use on microcomputers has been reviewed and evaluated by consortia and/or publishers. However, it is advisable to determine how these evaluations were conducted and what criteria were used. In some cases, such evaluations have been done by those having a vested interest in promoting the sale of such courseware. The guidelines provided in this article will facilitate such cross checking.

Each person tends to believe his/her definitions of quality related to microcomputer courseware agree with those held by other professionals. This is not necessarily so, hence, there is a need to compare and share evaluations among professionals to check for agreement among observations. Such comparisons usually lead to discussions about not only the quality of courseware but also the way in which microcomputer courseware can be integrated into the overall curriculum. This type of discussion should be encouraged because it tends to increase communication among staff about the overall quality of educational computing.

REFERENCES

Nelson, M. and Trueblood, C. "Field Based Delivery System For Rural Teachers. *The Rural Educator*. Vol. 6, No. 1, Fall 1984, pp. 9-11.

Trueblood, C. and Flanagan, K. "A Collaborative Effort for Staff Development." *The Rural Educator*. Vol. 5, No. 2, Winter 1983-84, pp. 27-32.

THE CONSTITUTION OF THE PENNSYLVANIA COUNCIL OF TEACHERS OF MATHEMATICS SEPTEMBER, 1985

ARTICLE I — NAME AND PURPOSE

Section 1. This organization shall be known as the Pennsylvania Council of Teachers of Mathematics, Inc. It is affiliated with the National Council of Teachers of Mathematics.

Section 2. The purpose of this council shall be:

1. To create and maintain an active interest in mathematics and encourage effective mathematics teaching at all levels in schools and colleges and universities.
2. To provide a medium for exchange of ideas and information pertaining to mathematics and to the teaching of mathematics.
3. To encourage research in mathematics and its teachings.
4. To facilitate mathematics activities and offer leadership to all who are concerned with mathematics education in the state.

ARTICLE II — MEMBERSHIP

Section 1. Membership shall be open to all persons who are engaged in the teaching of mathematics in public, independent, private, and parochial educational institutions and to persons who are interested in mathematics.

Section 2. Student membership shall be open to all students who are interested in mathematics and who are enrolled in any of the recognized institutions of higher learning in the state and have never taught professionally.

Section 3. Retired members of the Pennsylvania Council of Teachers of Mathematics, Inc. who have been in good standing for (5) successive years are eligible for life membership, with full membership privileges. They may continue to be members without payment of dues for as long as they signify a desire for membership.

Section 4. Institutional memberships shall be available to departments, libraries, schools, school districts, and other institutions. Such memberships exclude voting privileges.

ARTICLE III — DUES

Section 1. The dues will be specified by the By-Laws.

Section 2. The fiscal year of this council shall be from March 1st through February 28th.

Section 3. Dues Period

1. Dues shall coincide with the fiscal year.
2. Membership renewals shall be sent to all members in April.
3. Members whose renewals have not been received by June 1st, shall be notified.
4. Members whose dues are not paid by September 1st of the fiscal year shall be dropped from active membership.

ARTICLE IV — OFFICERS AND EXECUTIVE BOARD

Section 1. Officers

1. The officers of this council shall be President, Vice President, Secretary, and Treasurer.
2. The Vice President in alternate years will be President-Elect and Immediate Past President respectively: e.g., the person elected in 1977 as President-Elect will serve as Vice President during 1977-1978 and then as President from 1978 to 1980 and then again as Vice President during 1980-1981.

Section 2. Executive Board

1. The executive board of this council shall consist of the officers, four delegates-at-large, and the two representatives from each PCTM affiliated group. The president may, with board approval, appoint any PCTM member in good standing as NCTM representative, with voice and vote on board matters. Each chairman for both standing and ad hoc committees,

which must be approved by the executive board, shall be appointed by the president and will have voice and vote on board matters.
2. Any non-board member may attend the meetings of the executive board without the privilege of vote.
3. Quorum — A simple majority of the executive board shall constitute a quorum for the transaction of business. Two of those present must be officers.

Section 3. Qualifications
1. All members of the executive board shall be members of PCTM.
2. The officers of this council shall be members of the National Council of Teachers of Mathematics.
3. The President-Elect must have served as a member of the executive board, or as a committee chairman, or as NCTM representative for a period of at least one year, within a two year period prior to nomination.
4. The four delegates-at-large shall represent areas of the state not represented by PCTM affiliate groups.

Section 4. Term of Office
1. Officers — The term of office for all officers shall be two (2) years. The President shall not be eligible for re-election. The Vice President shall serve one year as President-Elect and one year as Immediate Past President.
2. Executive Board — The term of office for all members shall be two (2) years. They will be eligible for reappointment or re-election.
3. The term of office for executive board members shall commence on June 1. The outgoing officers shall complete their business by July 1.

Section 5. Elections and Appointments
1. The election of officers and delegates-at-large shall be by mail ballot. Delegates-at-large shall be elected by PCTM members who are not members of an affiliated group of PCTM. The ballot shall be prepared by the Committee on Nominations and Elections and mailed to the membership 45 days prior to the annual meeting. The ballots shall be returned to the Committee on Nominations and Elections 15 days prior to the annual meeting. Results of the election shall be made known at the annual meeting.
2. The committee on Nominations and Elections shall prepare a slate of at least two (2) nominees for each office. Ballots shall provide a place for write-in nominees.
3. Elections of officers and delegates-at-large shall be by plurality vote.
4. In case of a vacancy of an elected position on the executive board, the President shall appoint a member to fill the unexpired term. Such member must be in good standing.
5. The President-Elect, Secretary, and two delegates-at-large shall be elected in the odd years. One representative from each PCTM affiliated group shall be appointed in the odd years. All other board members shall be elected or appointed in even years.
6. Presidential appointments shall be reviewed by the executive board annually.

ARTICLE V — COMMITTEES

Section 1. The following are standing committees: Contest, Membership, Programs and Meetings, Publications, and Public Relations.

Section 2. Ad Hoc committees are Audit, Budget, Nominations and Elections, and such other committees as the work of the council might demand.

ARTICLE VI — MEETINGS

Section 1. There will be an annual meeting at which the business of the council will be conducted. There will also be a meeting of the executive board to approve the budget and conduct any other business of the council.

ARTICLE VII — AFFILIATION

Section 1. The Pennsylvania Council of Teachers of Mathematics, Inc. shall encourage the establishment of local mathematics groups inviting them to affiliate with the Pennsylvania Council of Teachers of Mathematics, Inc. The annual fee for local groups affiliating with the council shall be specified in the By-Laws.

ARTICLE VIII — DISSOLUTION AND NON-PROFIT STATUS

Section 1. If, at any time, this council shall cease to carry out the purpose as hereinstated, all assets and property held by it, whether in trust or otherwise, shall, after payment of its liabilities, be paid over to an organization, selected by the final executive board of this council, which has a similar purpose, and such assets and property shall be applied exclusively for such charitable, scientific, and educational programs.

ARTICLE IX — AMENDMENTS

Section 1. This constitution may be amended by vote of two thirds (⅔) of the members present at the annual meeting provided that the executive board has given written notice of the proposed amendment (or amendments) to the membership at least thirty (30) days prior to the annual meeting.

BY-LAWS

ARTICLE I — DUES

Section 1. All members as defined in Article II, Section 1, Section 2, and Section 4 shall pay annual dues which are set by the executive board and approved by the members attending the annual business meeting.

Section 2. All members as defined in Article II, Section 3 of the constitution shall be exempt from paying any dues.

ARTICLE II — DUTIES OF THE OFFICERS

Section 1. President
The President shall preside at all meetings of the council and shall serve as chairperson of the executive board; shall make appointments with the approval of the executive board; shall appoint members to fill vacancies of elected positions on the executive board; shall prepare an annual report of the activities of the council. The President shall be a member ex officio of all committees except of the Committee on Nominations and Elections.

Section 2. Vice President
The Vice President shall perform the duties of the president during the President's absence or in case of the President's inability to serve.

Section 3. Secretary
The Secretary shall record the minutes of each meeting of the council and of the executive board, and shall carry on the correspondence of the council.

Section 4. Treasurer
The Treasurer shall receive all money paid to the council; shall make all disbursements with the approval of the President; shall keep an accurate record of all receipts and expenditures; shall report to the membership chairperson an accurate list of names and addresses of all those who have paid their dues; shall submit records to be audited by the Auditing Committee. The Treasurer shall serve as chairperson of the Budget Committee.

Section 5. Executive Board
The executive board shall act as the governing body of the council; shall have the power to transact the business of the council; shall initiate, develop, and determine the policies of the council. The executive board shall have its annual meeting sometime previous to the annual meeting of the council, and shall meet at such other times as the President, or five members of the executive board shall deem necessary.

ARTICLE III — COMMITTEES

Section 1. Standing Committees

1. Contest
   The contest committee shall be responsible for conducting statewide student mathematics contests.
2. Membership
   The membership committee shall be responsible for bringing the purposes and advantages of membership in the council to the attention of mathematics teachers and to enlist their support of the programs of the council. The committee shall maintain an accurate list of current members. They shall cooperate with the affiliated groups to coordinate and maintain membership records for the affiliated groups.
3. Programs and Meetings
   The programs and meetings committee shall be responsible for planning and coordinating statewide and NCTM meetings.
4. Publications
   The publications committee shall be responsible for publishing a professional journal semi-annually and shall prepare other publications as the executive board deems necessary.
5. Public Relations
   The public relations committee shall be responsible for collecting and disseminating information and publicity of the council and its activities to the various districts in the state. The committee shall use the various forms of mass media to publicize the council and its work. They shall promote interests of mathematics education to effect change at the state and national levels.

Section 2. Ad Hoc Committees

1. Auditing
   In advance of the annual meeting, the President shall appoint an auditing committee of three (3) to audit the accounts of the Treasurer.
2. Budget
   The Treasurer and two members of the executive board shall constitute the budget committee. It shall be the duty of this committee to prepare an annual budget of the council and present it to the executive board at its annual meeting.
3. Nominations and Elections
   The nominations and elections committee shall be appointed by October 1. It shall consist of five (5) members: The Vice President who shall serve as chairperson; two (2) members from the executive board: two (2) members from the active membership outside of the executive board.

ARTICLE IV — AFFILIATIONS

Section 1. Newly affiliated groups are not required to pay dues during the school year in which they receive their charter.

Section 2. Other affiliated groups pay annual dues according to the following schedule:

Less than 201 members	$10.00
201 to 400 members	$20.00
More than 400 members	$30.00

Dues shall be paid to PCTM. The Treasurer of each group shall remit the necessary dues together with a completed form which will be sent by PCTM. The money from these dues is used to help defray costs related to affiliated group programs.

ARTICLE V — PARLIAMENTARY PROCEDURE

Section 1. *Robert's Rules of Order Newly Revised* shall be the authority in questions of parliamentary procedure.

CONSTITUTIONAL COMMITTEES

1. *Original Committee:*
   Constitution adopted — March 29, 1952
2. *First Revision Committee:*
   Revision adopted — April 1960
3. *Second Revision Committee:*
   Revision adopted — May 1, 1965
4. *Vote Adoption of Article VIII —*
   Non-Profit Status — May 10, 1969
5. *Third Revision Committee:*
   Revision adopted — May 11, 1974
6. *Fourth Revision Committee:*
   Revision adopted — March 26, 1976
7. *Vote Adoption of Name and Elections*
   Revision adopted — March 1981
8. *Vote Adoption of Officers and Terms of Office*
   Revision adopted — March 19, 1982
9. *Vote Adoption of Dues and Fiscal Year*
   Revision adopted — March 15, 1985